Risk Sharing versus Incentives Revisited

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Abstract

We extend the standard risk model in agency theory by introducing a population of agents with diverse risk preferences, while we still assume that principals are risk neutral. We show that in such an economy a stable equilibrium exists. More importantly, the predictions of our model are in contrast with the predictions derived when one assumes a homogeneous population of risk averse agents. This may provide an explanation as to why many empirical studies have failed to support the standard risk model. Our model yields new testable implications.

(Preliminary version)

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1. Introduction

One of the recent puzzles in economics is the inability of the risk model in agency theory to match the empirical data. In a principal-agent model with risk and risk averse players an optimal contract should balance incentives with risk sharing considerations efficiently. As the risk in the environment increases risk sharing becomes more important than the provision of incentives and vice versa. This prediction seems quite simple and compelling yet it lacks a strong empirical support. This paper attempts to reconcile the theory with the empirical evidence by looking at the principal-agent market rather than a given fixed principal-agent pair. We show that, in an economy where the agents have heterogeneous risk tastes, the risk model when analyzed from the "economy-wide principal-agent market" perspective may yield radically different predictions than when a fixed principal-agent pair is studied. To put it differently, the degree of risk aversion in a given contractual agreement is not exogenously given as most of the literature has assumed, but rather it is an outcome of an endogenous matching process.

A situation where the owner of a physical asset hires an agent to manage it and produce output is very common in economics, e.g. landlord-tenant in agriculture, timber sales, gold mining, oil and gas leasing, franchise and patent licensing. The method by which the agent is compensated for his input(s) into the productive process determines the level of output and hence the level of efficiency. A contract which provides the highest incentives is a fixed-rent contract which gives the entire output to the agent who in turn makes a fixed payment to the principal. For a long time this thought to be the most efficient arrangement. Adam Smith and Alfred Marshal criticized the practice of sharecropping in agriculture, where the agent (tenant) receives only a fraction of the output, on the basis that it does not provide full incentives to the tenants thereby lowering the level of production. However, with risk averse tenants sharecropping emerges as an efficient institution as it balances the incentives with the risk sharing [e.g. Cheung (1969) and Stiglitz (1974)], since a fixed-rent contract forces the risk averse agent to bear the entire risk while the risk neutral principal gets a riskless fixed payment. Since risk is present in most economic activities and the participants are usually risk averse one would expect this risk-based model to perform well empirically. Nevertheless, this is not the case.

Rao (1971) found that in India crops with high variability were more likely *not to be* sharecropped, casting doubt on the risk model which would have predicted the opposite. Allen and Lueck (1999), using individual contract data from more than 4000 contracts in modern North America agriculture did not find support for the risk model either. For a more detailed account of the empirical as well as the theoretical papers on this issue we refer the reader to the excellent surveys of the literature in Allen and Lueck (1995, 1999). On the other hand, Higgs (1973) utilizing state level data from the 1910 American South concluded that risk plays a major role in determining the nature of the tenurial contracts, which when combined with the above mentioned papers leads to the conclusion that the available evidence is at best mixed. Not only does not the risk sharing model fare well in agriculture, but its performance is problematic in other economic activities too [e.g. gold mining, natural gas and timber sales].[1]

[1] See [Allen and Lueck (1995), Table 1, p. 449], for empirical studies and their results on this issue outside

Most of the theoretical models on this issue assume that the agents have identical risk preferences [e.g. Stiglitz (1974), p.243].[2] This assumption is restrictive and as we show it may be one of the reasons that the empirical models do not perform satisfactorily. What is needed for the standard risk model's predictions to go through even in the presence of heterogeneous agents is the implicit assumption that any matching between the principals and the agents is random and not responsive to any changes in the parameters. Clearly these assumptions are not very plausible. We formulate a model with agents who have diverse attitudes toward risk. Principals are risk neutral and each one's asset is subject to a different exogenous variability. Each principal would like to match with the least risk averse agent as in this case incentives will be at the forefront and hence the principal's profit the highest. But the least risk averse agent goes to the principal with the highest bid (willingness to pay); the second lowest risk averse agent goes to the principal with the highest bid among the remaining principals and so on. We offer an algorithm which reaches a *stable equilibrium* and we show it exists. In such an equilibrium a "high variance principal" may be offering more incentives (relative to risk sharing) than a "low variance principal" which is in direct contrast with the risk sharing model when it is viewed from the perspective of a random matching and of a fixed principal-agent pair. Moreover, an increase in the variance of the asset of a principal may increase the probability of him offering a fixed-rent contract which again contradicts the standard risk model's premise. *These seemingly incompatible with the standard risk model predictions are in fact an outcome of this exact model which has been analyzed from a much broader and more realistic angle.* Therefore, omitting an analysis of the principal-agent market may lead to misleading conclusions.

A notable exception to the path that the literature has followed is the empirical work by Ackerberg and Botticini (2000) where, in a similar setting as ours, they employ the endogenous matching equilibrium concept as well.[3] They show that failing to control for endogenous matching may result in biased coefficients of interest. Using a historical dataset on agricultural contracts from Renaissance Tuscany, they find that tenants' risk aversion has played a role in contract choice. Our paper complements theirs by focusing primarily on the theoretical side of the matching process and delving more into how this matching takes place.

The analysis in our paper is designed to follow the landlord tenant paradigm in agriculture, although our model can be used to study any other similar market. Our reference point to the empirical literature is mainly Allen and Lueck (1999) because, in our opinion, it constitutes one of the most important recent studies which employs very detailed micro level data[4] and carefully tests the validity of the theoretical models. Of course their analysis and approach is perfectly valid when certain conditions are met. Our aim in this paper is precisely to identify those conditions [see section 4.3] and offer a theory which would serve

agriculture.

[2] In fact, Stiglitz (1974) in the first part of his paper (where the supply of labor is inelastic and hence only the risk sharing aspect of the problem is relevant) acknowledges that with diverse agents a condition for the stability of an equilibrium is needed [Equilibrium condition (a) and (b), p.222]. However, in the second part of his paper, where the labor supply is elastic and most of the empirical tests are based on the results derived therein, he assumes identical agents.

[3] This paper provides also a very extensive list of references on the issue.

[4] Leffler and Rucker (1991) is another paper which uses individual data (Timber-Harvesting contracts).

as a basis for empirical tests in a much broader setting.

The rest of the paper is organized as follows. Section 2 presents the model and gives the definition of a stable matching. Section 3 contains the main analysis. We begin this section by solving for the equilibrium share in a fixed landlord-tenant pair. Then, we examine a market which consists of two landlords and two tenants and point out the new predictions that our approach yields. Next, we consider a general market with n landlords and n tenants and offer three illustrative examples. Finally, we provide a system of equations which are consistent with our theory and could be used to test the validity of the risk model. Section 4 extends the model to include certain rigidities [e.g. "transportation costs"] into the market and the possibility of a fixed-rent contract. Furthermore, it compares the likelihood of a share contract with a fixed-rent one and provides a short discussion. We end in section 5 with some concluding remarks.

2. The model

We consider an agricultural economy which consists of L landlords indexed by ℓ ($\ell \in \{1, \ldots, L\}$) and T tenants indexed by t ($t \in \{1, \ldots, T\}$). We assume that $T = L$[5] and that each landlord has to hire a tenant to cultivate his land. Each landlord ℓ is paired with one tenant t to form a contractual relationship where the landlord provides the land and the tenant the skill and the effort in order to produce output. Let's denote a given pair by (ℓ, t). The production function for this specific pair is $y_{\ell,t} = e_t - \varepsilon_\ell$, where e_t denotes the effort that the tenant exerts, ε_ℓ is a random shock which is distributed normally with mean zero and variance σ_ℓ^2 and $y_{\ell,t}$ is the output produced. Without loss of generality, we normalize the per unit price of the output to one. Notice that the variance of the random component is not the same across landlords but it depends on the land.[6] Effort is costly, with its cost being $C(e) = e_t^2/2$. Landlord ℓ employs tenant t by offering him a wage $w_{\ell,t}$. We assume that all the tenants are risk averse, while the landlords are risk neutral.[7] Tenant t's utility function is given by $V = -\exp[-r_t(w_{\ell,t} - C(e_t)]$, where $r_t \geq 0$ is tenant t's degree of absolute risk aversion, which varies across tenants.[8] As Prendergast (1999) points out, in this setting, Holmstrom and Milgrom (1987) showed that the optimal compensation

[5] Since our focus is primarily on the effects of risk on the nature of tenurial contracts we would like to abstain from situations where there is competition for tenants ($L > T$) or when there is competition for landlords ($L < T$) as these extra considerations would contaminate our results.

[6] One can think of each land being suited for a different crop, or the same crop is grown in each land but the farms are geographically dispersed. In both cases, one can expect the effect of the exogenous conditions (weather, pest population) on the crop yield to be quite different from farm to farm. This is clearly the case in Allen and Lueck (1999) where each of the crop they examined (in the same region) had a different yield variability and the same crop had also a different yield variability from one region to another.

[7] This is a standard assumption in principal agent models applied to agriculture (e.g. Cheung (1969), Higgs (1973), Stiglitz (1974) and Allen and Lueck (1999)).

[8] As in Allen and Lueck (1999), we will assume declining absolute risk aversion. That is, as tenant t's wealth increases r_t decreases, i.e., the tenant becomes less risk averse. In the above mentioned paper there is clear evidence that there is a significant variation in the wealth of the farmers [see section 3.4]. Therefore, the assumption that the degree of risk aversion varies among farmers is very realistic.

scheme is linear and is given by $w_{\ell,t} = \beta_{\ell,t} + \alpha_{\ell,t} y_{\ell,t}$, where $\beta_{\ell,t}$ is tenant t's salary and $\alpha_{\ell,t}$ ($0 \leq \alpha_{\ell,t} \leq 1$) is his share of output.[9] We further assume that each tenant's reservation utility is U, each landlord's reservation utility is zero and that the landlords have all the bargaining power.[10] Our qualitative results would hold even if we had adopted more general utility, cost and production functions. The advantage with the parametric example is that it makes the exposition very transparent (since we are able to obtain closed form solutions) without loosing any important aspect of the analysis and it also provides the framework for econometric testing.

The game we consider unfolds as follows.

Stage 1. Each landlord is being matched with exactly one tenant. We require the matching to satisfy the notions of individual rationality and stability which are defined below.[11]

Definition 1 *A matching is individually rational if each pair provides to both parties more than their respective reservation utilities.*

Consider a given matching where there exists a landlord ℓ and a tenant t who are not matched to each other but who prefer each other to their current matching. Then, we say that the pair (ℓ, t) *blocks* the initial matching.

Definition 2 *A matching is stable if it is not blocked by any individual or a pair of players.*

Stage 2. In any given pair (ℓ, t) the landlord maximizes his expected utility by choosing the salary (or rent) $\beta_{\ell,t}$ and the share of output $\alpha_{\ell,t}$ that the tenant keeps.

Stage 3. Given his compensation the tenant maximizes his expected utility by choosing the level of effort $e_{\ell,t}$.

The random variable $\varepsilon_{\ell,t}$ is realized after all the decisions have been made. We look for a subgame perfect equilibrium of this game which satisfies individual rationality and stability.

[9] For an application of this model to labor contracts see Prendergast (1999).

[10] Since there are many landlords and tenants the reservation utilities we refer to are what the players receive when they do not participate in this market, i.e., a landlord may choose not to hire a tenant and a tenant not to work for a landlord.

[11] The definitions below have been taken from [Roth and Sotomayor (1990), pp. 20-21].

3. Analysis

First, we solve tenant t's problem who is employed by landlord ℓ. The tenant chooses his level of effort (given his wage) which maximizes his expected utility. That is,

$$\max_{e_t} EV = E[-\exp[-r_t(\beta_{\ell,t} + \alpha_{\ell,t}(e_t - \varepsilon_\ell) - e_t^2/2)]].$$

The above maximization problem is equivalent to

$$\min_{e_t} E[-V] = E[\exp[-r_t(\beta_{\ell,t} + \alpha_{\ell,t}(e_t - \varepsilon_\ell) - e_t^2/2)]]. \tag{1}$$

Taking the expectation of $-V$ with respect to the random variable ε_ℓ we obtain,

$$\begin{aligned} E[-V] &= \exp[-r_t\beta_{\ell,t} - r_t\alpha_{\ell,t}e_t + r_te_t^2/2]\ E[\exp[r_t\alpha_{\ell,t}\varepsilon_\ell]] \\ &= \exp[-r_t\beta_{\ell,t} - r_t\alpha_{\ell,t}e_t + r_te_t^2/2 + r_t^2\alpha_{\ell,t}^2\sigma_\ell^2/2], \end{aligned} \tag{2}$$

where the second equality follows from the fact that

$$\int_{-\infty}^{\infty} \exp[r_t\alpha_{\ell,t}\varepsilon_\ell] \exp[-\varepsilon_\ell^2/2\sigma_\ell^2](\sqrt{2\pi})^{-1} d\varepsilon_\ell = \exp[r_t^2\alpha_{\ell,t}^2\sigma_\ell^2/2].$$

By differentiating Eq.(2) with respect to e_t setting it equal to zero and solving for e_t we obtain,

$$e_t^* = \alpha_{\ell,t}. \tag{3}$$

Thus, as expected, the tenant's effort is in direct relationship with the share of output he gets to keep. Obviously, a fixed-rent contract (where $\alpha = 1$) provides the highest incentives for work. A fixed-rent contract however does not share the risk efficiently among the landlord and the tenant as the risk neutral landlord bears no risk which is born entirely by the risk averse tenant. An optimal contract would balance the incentives with the risk sharing appropriately. By plugging Eq.(3) into Eq.(2) we obtain,

$$E[-V]^* = \exp[-r_t\beta_{\ell,t} - r_t\alpha_{\ell,t}^2/2 + r_t^2\alpha_{\ell,t}^2\sigma_\ell^2/2].$$

Now let's turn to landlord ℓ's problem. By assumption landlords have all the bargaining power. Therefore the landlord will choose the tenant's compensation such that the tenant gets exactly his reservation utility. That is, $E[-V]^* = U$. By taking logs on both sides and solving for $\beta_{\ell,t}$ we obtain,

$$\beta_{\ell,t} = \frac{-\log U}{r_t} - \frac{\alpha_{\ell,t}^2}{2} + \frac{r_t\alpha_{\ell,t}^2\sigma_\ell^2}{2},$$

which when substituted into the landlord's expected profit function $\Pi = (1-\alpha_{\ell,t})e_t^* - \beta_{\ell,t}$ yields,

$$\Pi = \alpha_{\ell,t} - \frac{\alpha_{\ell,t}^2}{2} - \frac{r_t\alpha_{\ell,t}^2\sigma_\ell^2}{2} + \frac{\log U}{r_t}. \tag{4}$$

To simplify the analysis, we assume that $\log U = 0$, i.e., $U = 1$. The solution to the first order condition of Eq.(4) is,

$$\alpha^*_{\ell,t} = \frac{1}{1 + r_t \sigma^2_\ell}. \tag{5}$$

The comparative statics based on Eq.(5) indicate that as the variance increases the share decreases, while as the degree of absolute risk aversion decreases the share increases. These results are intuitively clear and well established in the theoretical literature on principal-agent models under risk. They are exactly the above predictions that are being tested by [Allen and Lueck (1999), p.707]. To be more precise, one of their regressions is based on $\alpha_{ij} = \sigma^2_{ij}\gamma_j + Z_{ij}\xi_j + \mu_j$ [Allen and Lueck (1999), Eq.(6), p.718], where σ^2 is the variance of the exogenous risk, Z is a vector of control variables (including the wealth of the tenant) and i and j index the contracts and the crops respectively. What the authors expect to find is a negative sign for the estimate of γ which is clearly the correct expectation given our analysis so far and Eq.(5). The relationship portrayed by Eq.(5) presumes the pair (ℓ, t) to be fixed which may not be a very realistic assumption. In a given agricultural area there are usually several landlords and tenants with different characteristics and quite a few crops. To give a preview of our subsequent analysis consider a pair (ℓ, t) and suppose that the variance of this crop increases. The increase in the variability of the exogenous factor may render the given pair unstable. Once the stability is restored (next we show how) the original landlord may have employed a new tenant whose degree of absolute risk aversion is much lower than that of the previous one and the resulting share may increase rather than decrease contradicting Eq.(5). Does that mean that the risk model is not valid? Clearly not. Simply the theoretical models so far have mainly focused on a specific landlord-tenant matching overlooking the mechanism which leads to this relationship.

Next, we study explicitly how the landlord-tenant market reaches a stable matching. For expositional purposes, we begin by assuming that there are only two landlords and two tenants.

3.1. A market with two landlords and two tenants

We begin by assuming that no pairs have been formed yet. Each landlord calculates his expected profit which result from each matching. In our 2×2 case each landlord has two numbers in his mind. His profit if he employes the first tenant and his profit if he employes the second one. We assume that the second tenant is more risk averse that the first one, i.e., $r_2 > r_1$. The expected profit of landlord ℓ who has hired tenant t is obtained by substituting Eq.(5) into Eq.(4) and is given by,

$$\Pi^*_{\ell,t} = \frac{1}{2(1 + r_t \sigma^2_\ell)}. \tag{6}$$

Each landlord prefers to employ the tenant with the low degree of risk aversion. The relatively low risk averse tenant can tolerate more risk and therefore the landlord can increase the incentives for effort (α) which increases the expected output and the landlord's profit. The question which naturally arises is who ends up with the low risk averse tenant?

Landlord ℓ's maximum willingness to pay for the low risk averse tenant is the difference between his expected profit with the low risk averse tenant and the high risk averse one, i.e., $\Pi^*_{\ell,1} - \Pi^*_{\ell,2}$. This difference, which is denoted by $M_\ell^{(1-2)}$, is given by,

$$M_\ell^{(1-2)} = \frac{\sigma_\ell^2(r_2 - r_1)}{2(1 + r_1\sigma_\ell^2)(1 + r_2\sigma_\ell^2)} > 0. \tag{7}$$

Proposition 3 *If $M_1^{(1-2)} > M_2^{(1-2)}$, then landlord 1 hires the low risk averse tenant. In this case the tenant's compensation is his expected wage ($w_{1,1}$) plus $M_2^{(1-2)}$. The high risk averse tenant receives just his expected wage ($w_{2,2}$). If the reverse inequality holds, then landlord 2 ends up with the low risk averse tenant and pays him his expected wage ($w_{2,1}$) plus $M_1^{(1-2)}$. The high risk averse tenant receives just his expected wage ($w_{1,2}$). Furthermore, the matching described above is individually rational and stable.*

Proof. First suppose that $M_1^{(1-2)} > M_2^{(1-2)}$. Consider the following matching: Landlord 1 $\longleftrightarrow$ Tenant 1 and Landlord 2 $\longleftrightarrow$ Tenant 2. Individual rationality follows trivially since each player in this matching gets more than his reservation utility. To prove that it is stable we must show that there does not exist a pair of players which blocks the above matching. First it is easy to see that landlord 1 will not pay more for tenant 1 than the second landlord's maximum willingness to pay, $M_2^{(1-2)}$. Any increase in the payment (above $M_2^{(1-2)}$) reduces the landlord's expected profits without any benefit since the payment is a fixed transfer and therefore the incentives remain unaltered and moreover the probability of hiring tenant 1 is already one. Let's now turn to the stability of the matching.

a) Landlord 1 would never wish to match with tenant 2. This can be seen by comparing $\Pi^*_{1,1} - M_2^{(1-2)}$ with $\Pi^*_{1,2}$. Since $M_1^{(1-2)} > M_2^{(1-2)}$, it follows that $\Pi^*_{1,1} - M_2^{(1-2)} > \Pi^*_{1,2}$.

b) Landlord 2 would like to match with tenant 1, but the maximum he is willing to pay is $M_2^{(1-2)}$. Landlord 1 can afford this plus an ϵ and consequently tenant 1 would rather remain with landlord 1.

c) From b it is clear that tenant 1 does not wish to match with landlord 2.

d) Tenant 2 would like to match with landlord 1 and receive something more than U. However, landlord 1 would be indifferent between the two tenants if he paid $U + M_1^{(1-2)}$ for tenant 1. Since now he is paying less (i.e., $U + M_2^{(1-2)}$) he clearly prefers to stay with the first tenant.

The proof is similar when $M_1^{(1-2)} < M_2^{(1-2)}$. ■

The landlord with the highest willingness to pay bids more for the low risk averse tenant and employs him. The compensation of this tenant is a wage which guarantees him in expected terms his reservation utility plus the maximum willingness to pay of the other landlord, while the high risk averse tenant receives from the other landlord exactly his reservation utility.

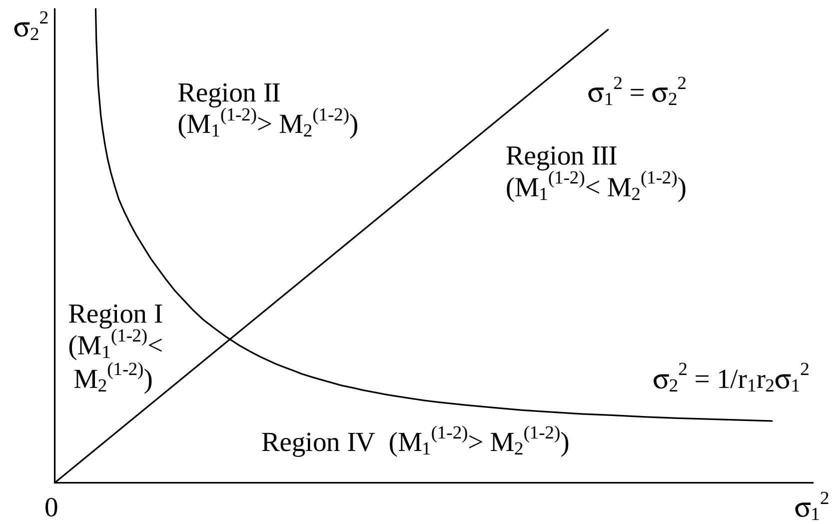

Figure 1:

3. 2. Comparative statics

It can be shown that $M_1^{(1-2)} > M_2^{(1-2)}$ if and only if,

$$(\sigma_1^2 - \sigma_2^2)(1 - r_1 r_2 \sigma_1^2 \sigma_2^2) > 0. \tag{8}$$

The inequality depicted in (8) is analyzed in figure 1. In regions I and III $M_1^{(1-2)} < M_2^{(1-2)}$, while in regions II and IV $M_1^{(1-2)} > M_2^{(1-2)}$. Therefore, if the exogenous parameters of our model fall in regions I or III landlord 1 ends up with the high risk averse tenant whereas landlord 2 hires the low risk averse tenant. The opposite holds when the parameters fall in regions II or IV. Moreover, the resulting matching in each case as we showed in the proposition above is stable. Surprisingly, the agent who is more tolerant toward risk does not necessarily match with the "high variance" principle [e.g. region II where the low risk averse tenant is hired by the landlord with the low variance].

Consider the basic regression equation $\alpha_{ij} = \sigma_{ij}^2 \gamma_j + Z_{ij}\xi_j + \mu_j$ we alluded to in section 3. What this relationship postulates is that as the variance increases, while controlling for other variables like wealth, the share that the tenant receives falls. This is also supported by our Eq.(5). Interestingly enough, this is half of the story. Suppose that the parameters $(r_1, r_2, \sigma_1^2, \sigma_2^2)$ are such that that we are in region I. The pairs in this case are Landlord 1 $\longleftrightarrow$ Tenant 2 and Landlord 2 $\longleftrightarrow$ Tenant 1. .Now let's focus our attention on the first pair. Then we should expect the relationship depicted by the regression equation above to hold. Assume that σ_1^2 increases. As long as we remain in region I the inverse relationship between the variance and the share holds [see Eq.(5)]. But the moment we move out of region I and enter region IV (or region II) the matching given above is no longer stable. When we enter into region IV the stable matching is Landlord 1 $\longleftrightarrow$ Tenant 1 and Landlord 2 $\longleftrightarrow$ Tenant 2. The variability of the exogenous factor (σ_1^2) has increased, but at the same time landlord 1 has switched from the high risk averse tenant to the low risk averse one. The net effect on the share is ambiguous. In fact, it may very well be the case that the share in that particular farm increases following the increase in the risk. Therefore, one cannot fix the contractual relationship and perform comparative statics as a change in some parameter may upset the entire market and its stability. Also notice that each landlord's willingness to pay for the low risk averse tenant, ceteris paribus, is high for moderate risk values and quite low for either high or low risk values. This is the reason why the low risk averse tenant does not necessarily match with the "high variance" landlord. This U-shape property carries over to the general case and we provide a further discussion in section 3.4. To illustrate our point better consider the following example.

Example 1. There are two landlords and two tenants. The parameters are $r_1 = 1/3$, $r_2 = 1$, $\sigma_1^2 = 1$ and $\sigma_2^2 = 1.2$. Tenant 2 is more risk averse than tenant 1 and the variability in farm 2 is greater than that in farm 1. Given the values of the parameters it can be easily calculated that we are in region I. This implies that landlord 2 is paired with tenant 1 whereas landlord 1 is paired with tenant 2. It can be calculated [from Eq.(5)] that $\alpha_{21}^* = .714$ with tenant 1 receiving a bonus (on the top of his reservation utility) equal to .125. In addition $\alpha_{12}^* = .50$ with tenant 2 receiving no extra bonus. Now suppose that σ_1^2 increases from 1 to 1.3. Then landlord 1 hires tenant 1 and the equilibrium share (in farm 1) rises as well to $\alpha_{11}^* = .70$ with tenant 1 receiving a bonus equal to .13.

Despite the fact that the variance in farm 1 has increased the landlord offers a much greater share of the output to the tenant. The only difference is that he has employed a new tenant who is more tolerant towards risk. The reason is that the increase in the variance increased landlord 1's willingness to pay for the low risk averse tenant which surpassed that of landlord 2. Therefore landlord 1 ends up biding more than landlord 2 for tenant 1 and hires him.

Laffontaine (1992) observes that if the franchisors (principals) have all the bargaining power the participation constraint of the franchisee (agent) is binding and therefore royalties and franchise fees should be negatively correlated. However, as she puts it "...observed royalty rates and franchise fees are not negatively or (positively) related..." Nonetheless, participation constraints need not be binding even when we begin by assuming that principals have all the bargaining power. This is clearly illustrated in the above 2×2 example where the low risk averse tenant's constraint is not binding, a result which carries over to the general case we study in the next section. Another interesting observation which our example brings forth is that if we consider just a random matching, then the farm with the highest variance should (on average) have the lowest share. This would indeed be the case in our example. Landlord 1 would have a 50% chance of being paired with tenant 1 and the same holds for landlord 2. The expected share in farm 1 would be .625 while in farm 2 it would be .584 (farm 1 has a lower variance than farm 2, i.e., $\sigma_1^2 = 1 < \sigma_2^2 = 1.2$). However this expectation is totally overturned if we instead eliminate the unstable matching (i.e., Landlord 2 $\longleftrightarrow$ Tenant 2 and Landlord 1 $\longleftrightarrow$ Tenant 1 is indeed unstable). In this case, the farm with the low variance (farm 1) has a share equal to .5, whereas the farm with the high variance (farm 2) has a share equal to .714. A researcher looking at such data may conclude that there is no support for the risk model when in fact these data are perfectly consistent with that model.

Our main point is that, in an economy where the agents have diverse attitudes toward risk, the equation $\alpha_{ij} = \sigma_{ij}^2\gamma_j + Z_{ij}\xi_j + \mu_j$ ignores the issue of the stability of matching and may lead to misleading results and conclusions. Essentially, it assumes two interrelated things: 1) The landlord and the tenant are paired randomly and 2) The contractual agreement is immune to any exogenous changes. Obviously these are extreme assumptions which are justified only if one introduces severe rigidities[12] into the market, like prohibitively high costs for a tenant of moving from one farm to another, or other types of preferences which are not captured by the present model. The extent to which these rigidities are present is an interesting and important empirical question. However, it seems more reasonable to start building a theoretical model by assuming away such rigidities which as long as they are modest they do not affect qualitatively our main argument.

Having established the main predictions of the 2×2 model we now turn to a general market which consists of n landlords and n tenants.

3.3. A market with n landlords and n tenants

We analyze the stability of an equilibrium and its properties in an $n \times n$ market. The only added difficulty in this case is that when a landlord wants to compute his maximum willingness to pay for tenant t, it is not immediately apparent what his opportunity cost is. In other words, who the alternative tenant is that he will hire in the event he does not hire tenant t. This is due to the presence of many landlords who are competing over many tenants. We provide an algorithm that overcomes this problem and then we show that in this market there exists a stable matching.[13]

Landlord ℓ's bidding strategy[14] is given by a vector,

$$B_\ell = (B_{\ell,1}, \dots B_{\ell,t}, \dots , B_{\ell,n}) \in R_n,$$

where the first coordinate indicates how much he is willing to pay to acquire tenant 1. Likewise, the last coordinate indicates how much he is willing to pay to acquire tenant n. We further assume that each tenant is hired by the highest bidder. If landlord ℓ has the highest bid for more than one tenant he hires the one with the lowest degree of risk aversion. A strategy profile for all landlords is $B = (B_1, \dots , B_\ell, \dots , B_n)$. A profile B^* constitutes an equilibrium when no landlord wishes to deviate by changing his strategy B_ℓ and become better off. An equilibrium bidding B^* yields an equilibrium matching between landlords and tenants.

[12] In section 4.1, we explore the ramifications when one plausible rigidity is introduced.

[13] The set-up is similar to the *marriage market* problem due to Gale and Shapley, which can be found in [Roth and Sotomayor (1990), p.27]. Our model is different from the marriage model in the sense that our players (landlords) have all the same preference ordering over the tenants but they differ in their willingness to pay. In the marriage market model men may not have the same preference ordering over women and there are no side payments.

[14] Since this is a model of complete information the bidding is imaginary. Each landlord knows what the others are willing to pay for each tenant and given that the tenant that he is going to hire. Therefore the landlord-tenant pairs are sorted out from the beginning without any formal auction taking place.

The algorithm proceeds as follows. First we assume that all n tenants have different attitudes towards risk and we rank them according to their degree of absolute risk aversion, $r_n > r_{n-1} > \ldots > r_2 > r_1$. As in the previous subsection all landlords are risk neutral. Now let's describe the landlords' preferences over tenants. Landlord ℓ's expected profit function is given by Eq.(6). Therefore as in the 2×2 case each landlord prefers to match with the first tenant, i.e., the one with the lowest degree of risk aversion. Hence landlord ℓ's profits are ranked as follows: $\Pi^*_{\ell,1} > \ldots > \Pi^*_{\ell,n}$, $\ell = 1, \cdots, n$.

Step 1. We find the difference in the landlords' profits between when hiring tenant n and $n-1$, i.e., $M_\ell^{((n-1),n)} = \Pi^*_{\ell,n-1} - \Pi^*_{\ell,n}$, $\ell = 1, \cdots, n$. Assuming that there are no ties we rank the $M_\ell^{((n-1),n)}$'s. The landlord with the highest M_ℓ places a bid slightly higher than the second highest maximum willingness to pay and wins tenant $n-1$ (the relatively lower risk averse tenant between the two so far). In case of a tie assume that the landlords flip an unbiased coin.

Step 2. We proceed by adding tenant $n-2$. So, we have the landlords competing over three tenants, i.e., n, $n-1$ and $n-2$.[15] Consider the same landlord ℓ who won in step 1 and suppose that he bids $B_{\ell,n-2}$ for tenant $n-2$ and wins. Is this an equilibrium? Since we know that landlord ℓ has won tenant $n-2$ we can also figure out (from the previous step) the tenants that the remaining landlords are hiring and their maximum willingness to pay for tenant $n-2$. Denote those bids by an $((n-1) \times 1)$ vector $M_{\ell_{-1},n-2}$ and rank them from the highest to the lowest. Since we supposed that landlord ℓ is winning tenant $n-2$ it must be the case that his bid is higher than the maximal element of the vector $M_{\ell_{-1},n-2}$. Denote this maximal element by $\widehat{M_{\ell_{-1},n-2}}$. Then landlord ℓ's profits if he wins (in this case he will bid $B_{\ell,n-2} = \widehat{M_{\ell_{-1},n-2}} + \epsilon$) are $\Pi^*_{\ell,n-2} - \widehat{M_{\ell_{-1},n-2}}$. If he does not win tenant $n-2$ the landlord who submitted $\widehat{M_{\ell_{-1},n-2}}$ does and landlord ℓ ends up with either tenant $n-1$ or tenant n. Thus, landlord ℓ can simply compare his payoff between winning tenant $n-2$ and not. When the former is greater than the latter step 2 ends and landlord ℓ wins tenant $n-2$. This is clearly an equilibrium so far since no landlord has an incentive to change his bid. Otherwise there is a new tentative winner (the landlord who submitted $\widehat{M_{\ell_{-1},n-2}}$) who faces the same decision problem that landlord ℓ did before.

Step 3. We repeat the same procedure as in step 2 by adding tenant $n-3$.

The algorithm ends with step $n-1$ when we add the last tenant (i.e., tenant 1).

Proposition 4 *A stable equilibrium in the $n \times n$ landlord-tenant market exists.*

Proof. See appendix. ■

To illustrate the algorithm and various properties of a stable equilibrium we provide the following three concrete examples where each one yields a different prediction.

[15] Of course in this truncated market only three landlords will eventually hire tenants. This is not so important at this point since now we are trying to determine the opportunity cost of each landlord hiring a specific tenant. At the end of the algorithm every landlord is paired with one tenant and vice versa.

Example 2. Consider a market with four landlords and four tenants. The tenants are ranked according to their degree of absolute risk aversion, $r_4 = 4 > r_3 = 3 > r_2 = 2 > r_1 = 1$. The variance in each farm is, $\sigma_1^2 = 3$, $\sigma_2^2 = 2$, $\sigma_3^2 = 5$ and $\sigma_4^2 = 4$. We can compute the expected profits [using Eq.(6)] of each pair (landlord, tenant) which are depicted below (landlords are in rows and tenants in columns),

	1	2	3	4
1	.125	.071	.05	.038
2	.166	.1	.071	.055
3	.083	.045	.031	.024
4	.1	.055	.038	.03

For example, cell 1,2 indicates that when landlord 1 is paired with tenant 2 the landlord's expected profit is .071.

Step 1. Compute the $M_\ell^{(3,4)}$'s and rank them. That yields, $M_2^{(3,4)} = .016$, $M_1^{(3,4)} = .012$, $M_4^{(3,4)} = .008$ and $B_3^{(3,4)} = .007$. Thus landlord 2 bids .012 and wins tenant 3.

Step 2. Now we add tenant 2 and consider the competition among the landlords for tenants 2,3 and 4. Begin by supposing that landlord 2 wins tenant 2. Then the opportunity cost of landlord 1 is tenant 3 (since he has the highest willingness to pay among the remaining landlords), while the opportunity costs of landlords 3 and 4 are tenant 4. It can be easily calculated that the maximum willingness to pay of landlord 1 for tenant 2 is .071-(.05-.008)=.029; the maximum willingness to pay of landlord 3 for tenant 2 is .0216; and the maximum willingness to pay of landlord 4 for tenant 2 is .025. Hence, landlord 1 has the highest maximum willingness to pay (besides the tentative winner who is landlord 2). Therefore, landlord 2 must bid at least .029 to win tenant 2. If he does that he enjoys a profit equal to .1-.029=.071. Otherwise, landlord 1 wins tenant 2, while landlord 2 hires tenant 3 and enjoys a profit equal to .071-.008=.063 $<$ .071. Notice that .008 is the second highest bid for tenant 3 after factoring in the fact that landlord 1 is the tentative winner. Thus, landlord 2 bids .029 and wins tenant 2.

Step 3. Now we add tenant 1 and consider the competition among the landlords for tenants 1,2,3 and 4. Begin by supposing again that landlord 2 wins. Then the opportunity cost of landlord 1 is tenant 2 (since landlord 2 wins tenant 1 from step 2 it can be deduced that landlord 1, who has the highest willingness to pay among the remaining landlords, wins tenant 2); the opportunity cost of landlord 4 is tenant 3; and the opportunity cost of landlords 3 is tenant 4. It can be easily calculated that the maximum willingness to pay of landlord 1 for tenant 1 is .125-(.071-.024)=.078; the maximum willingness to pay of landlord 4 for tenant 1 is .1-(.038-.007)=.069; and the maximum willingness to pay of landlord 3 for tenant 1 is .083-.024=.059. Hence, landlord 1 has the highest maximum willingness to pay (besides the tentative winner who is landlord 2). Therefore, landlord 2 must bid at least .078 to win tenant 1. If he does that he enjoys a profit equal to .166-.078=.088. Otherwise, landlord 1 wins tenant 1, while landlord 2 hires tenant 2 and enjoys a profit equal to .1-.024=.076 $<$ .088. Thus, landlord 2 bids .078 and wins tenant 1.

To summarize, the equilibrium matching is: Landlord 2 $\longleftrightarrow$ Tenant 1 and the landlord pays a bonus equal to .078 and offers a 33% share of the output; Landlord 1 $\longleftrightarrow$ Tenant 2 and the landlord pays a bonus equal to .024 and offers a 14% share of the output; Landlord 4 $\longleftrightarrow$ Tenant 3 and the landlord pays a bonus equal to .007 and offers an 8% share of the output and Landlord 3 $\longleftrightarrow$ Tenant 4 and the landlord pays no bonus and offers a 5% share of the output.

Example 3. Consider a market with three landlords and three tenants. The tenants are ranked according to their degree of absolute risk aversion, $r_3 = 3 > r_2 = 2 > r_1 = 1$. The variance in each farm is, $\sigma_1^2 = 1$, $\sigma_2^2 = .51$ and $\sigma_3^2 = .3$. We can compute the expected profits [using Eq.(6)] of each pair (landlord, tenant) which are depicted below (landlords are in rows and tenants in columns),

	1	2	3
1	.25	.166	.125
2	.331	.247	.197
3	.384	.3125	.263

Step 1. Compute the $\mathrm{M}_\ell^{(2,3)}$'s and rank them. That yields, $\mathrm{M}_2^{(2,3)} = .05$, $\mathrm{M}_3^{(2,3)} = .0495$ and $\mathrm{M}_1^{(2,3)} = .041$. Thus landlord 2 bids .0495 and wins tenant 2.

Step 2. Now we add tenant 1 and consider the competition among the landlords for tenants 1,2 and 3. Begin by supposing that landlord 2 wins tenant 1. Then the opportunity cost of landlord 3 is tenant 2 (since he has the highest willingness to pay among the remaining landlords), while the opportunity costs of landlord 1 is tenant 3. It can be easily calculated that the maximum willingness to pay of landlord 1 for tenant 1 is .25-.125=.125; the maximum willingness to pay of landlord 3 for tenant 1 is .384-(.3125-.041)=.1125. Hence, landlord 1 has the highest maximum willingness to pay (besides the tentative winner who is landlord 2). Therefore, landlord 2 must bid at least .125 to win tenant 1. If he does that he enjoys a profit equal to .331-.125=.206. Otherwise, landlord 1 wins tenant 1, while landlord 2 hires tenant 2 and enjoys a profit equal to .2475-.0495=.198 $<$ .206. Notice that .0495 is the second highest bid for tenant 2 after factoring in the fact that landlord 1 is the tentative winner of tenant 1. Thus, landlord 2 is better off employing tenant 1 and landlord 3 employs tenant 2. It can be easily checked that this is a stable equilibrium.

To summarize, the equilibrium matching is: Landlord 2 $\longleftrightarrow$ Tenant 1 and offers a share equal to 66%; Landlord 1 $\longleftrightarrow$ Tenant 3 and offers a share equal to 25% and Landlord 3 $\longleftrightarrow$ Tenant 2 and with share equal to 62.5%.

Example 4. Consider a market with three landlords and three tenants. The tenants are ranked according to their degree of absolute risk aversion, $r_3 = 3 > r_2 = 2 > r_1 = 1$. The variance in each farm is, $\sigma_1^2 = .1$, $\sigma_2^2 = .07$ and $\sigma_3^2 = .05$. We can compute the expected profits [using Eq.(6)] of each pair (landlord, tenant) which are depicted below (landlords are in rows and tenants in columns),

	1	2	3
1	.454	.417	.385
2	.467	.439	.413
3	.476	.454	.435

Following the same steps as in example 3 the stable equilibrium is: Landlord 1 $\longleftrightarrow$ Tenant 1 and offers a share equal to 91%; Landlord 2 $\longleftrightarrow$ Tenant 2 and offers a share equal to 88% and Landlord 3 $\longleftrightarrow$ Tenant 3 and with share equal to 87%.

In a market which consists of more than two landlords and two tenants, as in the 2×2 case analyzed in sections 3.1 and 3.2, the relationship between the exogenous variability and the share is not necessarily negative, i.e., higher variance farms offer lower share. In example 2 this relationship is indeed negative. However, in example 4 it is positive while in example 3 it is not even monotonic.[16] A regression of the exogenous variability on the share, in an agricultural economy like the one in example 4, would have yielded positive estimates, while in an economy like in example 3 would have probably yielded insignificant estimates, like most of the estimates in [Allen and Lueck (1999), table 5]. Without any consideration of the stability of matching these mixed results will force a researcher to falsely refute the risk model.

3.4. An econometric specification

We can conclude from the analysis in the previous sections that the degree of risk aversion of a farmer in a contractual relationship is not exogenously given but rather it is determined by the variance of the output in the specific farm. The important question which arises is what is the form of this relationship? We argue that this relationship is governed by a U-shape functional form [that is also true in the 2×2 model of section 3.2]. Fix the degree of risk aversion of all tenants in the economy and observe that for sufficiently low variance in a given farm the gain of the landlord from hiring a low risk averse tenant is low. Since risk is low risk sharing is not a major consideration. Hence incentives and output are relatively high no matter who the employed tenant is. The other extreme is when the variance is sufficiently high (relative to the degrees of risk aversion in the economy). In this case the willingness to pay for a low risk averse tenant is again low for the opposite reason than before. Risk sharing is very important and therefore the landlord's expected profits are quite low regardless of the risk preferences of the tenant. The highest willingness to pay for low risk averse tenants comes from farms with moderate levels of exogenous variability. Consider an agricultural economy with n farms (landlords) and n tenants where i is an index for the farm. Then, one functional form which depicts the relationship between the degree of risk aversion in farm i and its variance and captures the basic qualitative features of the theoretical analysis is,

$$r_i = A - B\sigma_i^2 + C(\sigma_i^2)^2. \tag{9}$$

The above equation[17] says that the degree of risk aversion of a tenant working in farm i is high when the variance in that farm is either very low or very high. As the variance

[16] If one views the matching as a random process, then the risk model should produce a negative relationship between the share and the variance.

[17] Of course this is only one canditate equation for this relationship. Clearly other functional forms which also exhibit a U-shape may fit the data better.

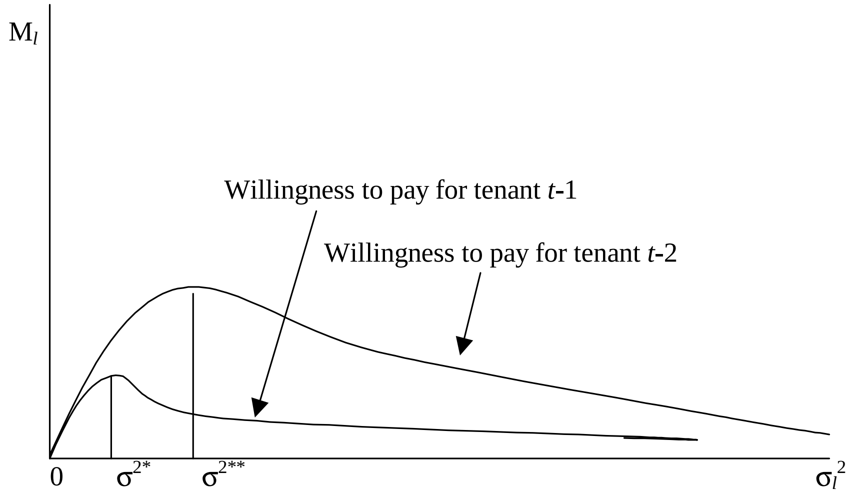

Figure 2:

attains values closer to the “average” the degree of risk aversion in that farm decreases, i.e., given our approach so far that means that a new tenant is hired who is more tolerant toward risk.

The argument we make in this section can be seen more rigorously if we extend the analysis of section 3.3. Consider an $n \times n$ market and its stable equilibrium. Pick one farm (ℓ) who has reached an agreement with tenant t. Landlord ℓ's maximum willingness to pay, as a function of σ_ℓ^2, for the tenant with the next lower degree of risk aversion (tenant $t-1$) is given by Eq.(7) (with the appropriate changes in the subscripts). The same holds for tenant $t-2$ and so on. Figure 2 presents the graphs of $M_\ell^{(t-1,t)}$ and $M_\ell^{(t-2,t)}$. Landlord ℓ's maximum willingness to pay for tenant $t-1$ attains its maximum at σ^{2*} while that for tenant $t-2$ at σ^{2**}. We can see that as the variance moves from the two extremes to inside the willingness to pay increases. Hence one should expect the r_i to be high in farms with either high or low variance and low in farms with “average” exposure to the exogenous forces. In example 2, where the variances in the economy are relatively high, the landlord-tenant pairs are positioned on the increasing part of the U-shape function, i.e., the landlord with the lowest variance hires the tenant with the lowest r and the landlord with the highest variance hires the tenant with the highest r. In example 4, where the variances are relatively low, the economy is located on the decreasing part of the U-shape function, while in example 3 (with moderate variances) it is located on the bottom of the function where the landlord with the “average” variance employs the least risk averse tenant.

Now consider Eq.(5) which postulates that the share is in inverse relationship with the degree of risk aversion and the variance. By substituting Eq.(9) into Eq.(5) we obtain,

$$\alpha_i = \frac{1}{1 + A\sigma_i^2 - B(\sigma_i^2)^2 + C(\sigma_i^2)^3}, \tag{10}$$

which indicates that α_i and σ_i^2 are not necessarily related in a negative or even a monotonic way as examples 2-4 have clearly pointed out. Moreover, one does not need data on wealth if he wishes to estimate the reduced form equation given by Eq.(10).

4. Extensions and discussion

We investigate an extension of the basic model presented so far into two different directions. One is the incorporation of some reasonable rigidities and the other is the comparison of the profitability of a fixed-rent contract with a sharecropping one.

4.1. Rigidities

One rigidity for example may be the fact that tenants are unwilling to accept a job in another "region," i.e., the cost associated with such an employment is very high which surely offsets any benefits. Allen and Lueck (1999) divide their data set into regions, parishes or counties and assume that the exogenous variability of each crop is the same within a certain region but it varies from one region to another. Of course within one region there are several crops which do not have identical variance. Depending on the size of the regions it may be reasonable to assume that tenants in one area may find it costly to manage a farm in a neighboring region. On the other hand, within the same area, it is fairly acceptable to assume that tenants are free from such constraints. We introduce such a rigidity in order to demonstrate how the model can be extended to handle this case and that our results of the previous section are robust.

Consider for instance an almost trivial extension of example 1. There are two regions, region A and region B each one having two landlords and two tenants and two crops, for concreteness wheat and corn. No tenant in region A wishes to work in region B and vice versa. However, the tenants in the same region can accept employment in either farm. The data in region A are the same as in example 1 with wheat being the crop grown in farm 1 and corn the one grown in farm 2. In region B the data are the same as in example 1 with the only difference that wheat is grown in farm 2 and corn in farm 1. Consider the farms in the two regions which grow wheat. Following the analysis after example 1, the share in the high variance farm (which is in region B) is .714, whereas the share in the low variance farm (which is in region A) is .5.[18] The high variance crop is cultivated under a contract which specifies a higher share than the low variance crop in the other region. This observation would not hold if the matching *within each region* was random. We can obtain similar results in the $n \times n$ market [following the logic of examples 2-4] all of which pointing to the fact that even in the presence of "transportation costs" the relationship between the share and the variance is not necessarily negative across regions (for the same crop) and across crops (in the same region).

[18] A similar finding is true for the other crop (corn).

4.2. Fixed-rent versus sharecropping

Another prediction of the risk-model that has often been tested empirically is that as the variance increases fixed-rent contracts become less likely. This rests on the assumption that share tenancy has certain disadvantages, like output underreporting [see Allen and Lueck (1999), p. 725], monitoring etc. Therefore, if the variance is relatively low the fixed cost of the share contract outweighs its risk sharing benefits and a fixed-rent contract is chosen. Otherwise, that is if the variance is high, a share contract is more likely to emerge as an equilibrium outcome. Or, in other words, farms with higher variance are more likely to offer share contracts than the ones with lower variance. It is this prediction that in Allen and Lueck (1999)[19] is not supported by the data. Below we present a simple 2×2 risk model which shows that in certain regions of the parameters the opposite prediction holds. That is, the farm with the high variance offers a fixed-rent contract while the low variance farm offers a share one. Table 3 in Allen and Lueck (1999) depicts more or less this relationship. Also Rao (1971) finds that relationships in farms with high variance are more likely to be governed by fixed-rent contracts. Does this mean that the contract choice is not based on risk considerations, or that some aspect of the risk model has been overlooked?

Consider the same model as in subsection 2.1, i.e., two landlords and two tenants with $r_2 > r_1$. Share tenancy has a fixed cost equal to k. Then, the profit of landlord ℓ who employs tenant t and offers a share contract is,

$$\Pi^{*,s}_{\ell,t} = \frac{1}{2(1 + r_t\sigma^2_\ell)} - k, \tag{11}$$

where the s superscript indicates that the offered contract is sharecropping. If landlord ℓ decides to lease his land on a fixed-rent basis he sets $\alpha_{\ell,t} = 1$ and his profit is [from Eq.(4)],

$$\Pi^{*,fr}_{\ell,t} = \frac{1 - r_t\sigma^2_\ell}{2}. \tag{12}$$

Clearly if $k = 0$ share tenancy is preferred by the landlord to a fixed-rent agreement.

Figure 3 depicts the range of the variance for which one leasing arrangement dominates the other. As it can be seen from the figure for any $\sigma_\ell < \sigma^*_t = (k + \sqrt{k(k+2)})/r_t$ fixed-rent contracts are preferred. When $\sigma_\ell > \sigma^*_t$ share cropping dominates. So far the analysis yields predictions similar to those which are tested in Allen and Lueck (1999), i.e., as the variance increases share tenancy becomes more likely. This prediction holds as long as we examine a fixed landlord-tenant pair. The moment we examine the whole (2×2 in this example) market we may end up with the exact reverse prediction even though the risk model is still intact.

We begin the analysis by observing that since $r_2 > r_1$ it must be that $\sigma^*_1 > \sigma^*_2$. This divides the space of the variances $(\sigma^2_1 \times \sigma^2_2)$[20] into three relevant regions which are depicted in figure 4. Region I, region II and region III.

[19] For the exact specification of the test see [Allen and Lueck (1999), page 715, Eq.(4) and (5)].

[20] Notice that the subscripts in σ^*_1 and σ^*_2 refer to the two tenants while the subscripts in σ^2_1 and σ^2_2 refer to the landlords.

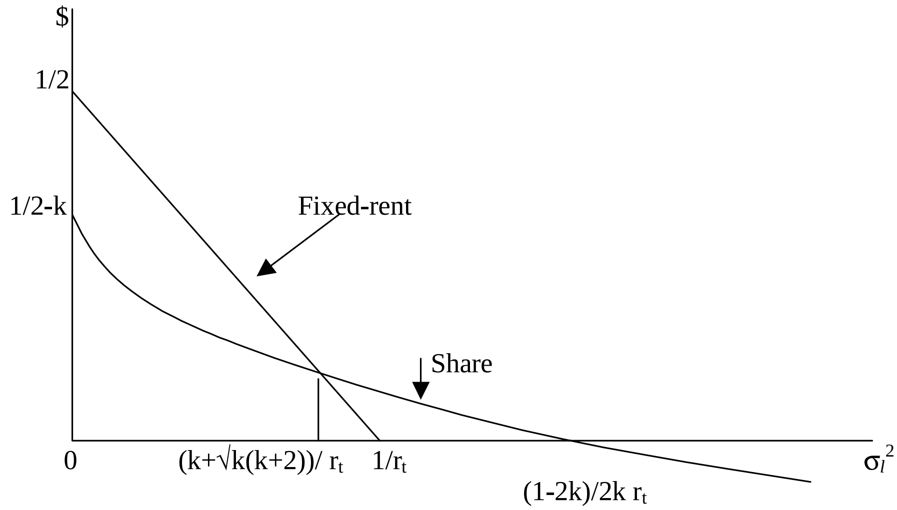

Figure 3:

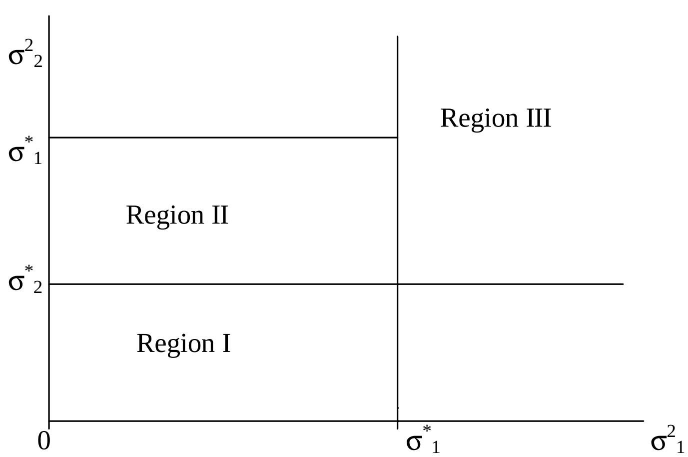

Figure 4:

Region I. In this case the variances are relatively low and both landlords offer fixed-rent contracts and their profits are given by Eq.(12). Therefore landlord ℓ's maximum willingness to pay for tenant 1 is,

$$M_\ell^{(1-2)} = \frac{\sigma_\ell (r_2 - r_1)}{2}.$$

Region II. The landlord who hires tenant 1 offers a fixed-rent contract (since $\sigma_\ell^2 < \sigma_1^*$) whereas the other landlord offers a share. Therefore landlord ℓ's maximum willingness to pay for tenant 1 is,

$$M_\ell^{(1-2)} = \frac{1 - r_1 \sigma_\ell^2}{2} - \frac{1}{2(1 + r_2 \sigma_\ell^2)} + k.$$

Region III. Both landlords offer share and the analysis is exactly the same as in the 2×2 model of section 3.1.

Consider region II. Landlord 1 wins tenant 1 if and only if $M_1^{(1-2)} > M_2^{(1-2)}$. This is equivalent to,

$$(\sigma_1^2 - \sigma_2^2)[(r_2 - r_1) - r_1 r_2(\sigma_1^2 + \sigma_2^2 + r_2 \sigma_1^2 \sigma_2^2)] > 0. \tag{13}$$

Further assume that $\sigma_1^2 > \sigma_2^2$. Is it possible in this economy to have the high variance landlord offering (systematically) a fixed-rent contract when at the same time the low variance landlord offers a share? This can indeed be the case if $(r_2 - r_1) - r_1 r_2(\sigma_1^2 + \sigma_2^2 + r_2 \sigma_1^2 \sigma_2^2) > 0$ and landlord 1 hires tenant 1 and offers him a fixed-rent. The next numerical example illustrates this point.

Example 5. Consider a market with two tenants and two landlords. The degree of risk aversion for tenant 1 is $r_1 = .001$ and for tenant 2 is $r_2 = 1$. The variance in landlord 1's farm is $\sigma_1^2 = 1$ while in landlord 2's is $\sigma_2^2 = .1$. The fixed cost of the share contract is $k = .002$. It can be easily checked that inequality (13) holds and the high variance landlord offers a fixed-rent while his low variance counterpart offers a share.

This observation would seem to contradict the standard risk model while it is an outcome of this model. Furthermore assume that the variance in farm 2 increases from .1 to (say) 1.1. In this farm before the increase in the variance the landlord was offering a sharecropping contract. The increase in the variance, according to the standard risk model, should lower the probability of a fixed-rent contract, or in a model like ours where there are no random errors one should never expect the landlord to offer a fixed-rent. However, the increase in the variance increases landlord 2's willingness to pay for tenant 1 and outbids landlord 1. Since landlord 2 has now employed tenant 1 he offers a fixed-rent contract.

What one can observe from the above analysis is that the prediction that as the variance increases share tenancy becomes more likely, or that farms with higher variance should on average offer more sharecropping contracts does not necessarily hold. If one, in a certain area, observes the coexistence of fixed and share contracts it must be the case that we are in region II[21] in figure 4 (otherwise we would have observed only fixed or share contracts). But the very moment we move into region II the situation described by example 3 may arise. Then it is not surprising at all that when one ignores the landlord-tenant market the empirical evidence does not seem to conform with the risk model. Consider for instance the table below which has been taken from [Allen and Lueck (1999), Table 3, p. 713].

	Crop variability	**Fraction of cropshare contracts**
Corn (irrigated)	.023	.58
Corn (dryland)	.14	.64
Soybeans	.143	.72
Oats	.191	.59
Sorghum	.195	.59
Barley	.238	.53
Wheat	.247	.61

The table depicts the relationship between crop variability (in South Dakota, 1975-1991) and the fraction of cropshare contracts. Corn (irrigated), for example, with very low variance is produced under more sharecropped contracts (58%) than barley (53%) which has a much higher variance. However, as we explained above, this finding may be perfectly consistent with the risk model.

We can apply the algorithm outlined in section 3.3 to analyze the choice between a fixed-rent and a cropshare contract in an $n \times n$ market. The only difference is that now in each step a landlord will have to choose between the two types of contracts. Existence of a stable equilibrium is easily established by applying proposition 4.

4.3. Discussion

Our model would be indistinguishable from the standard risk one if all tenants had the exact same degree of risk aversion. In that case the assumption that the matching is random and untouched by external forces would be innocuous. The presence of enough variability in the tenants' characteristics however does not warranty making such an assumption. If one has contract data from an entire state (or states) such variability will almost certainly appear. Dividing a state, for example, into smaller areas and assuming enough homogeneity (in terms of risk aversion) within each area is a good starting point. If on the top of that we assume that tenants do not accept employment outside their area, then we have the set of conditions needed to justify the empirical framework which has been adopted in the literature so far [e.g. Allen and Lueck (1999)]. Otherwise, that is if these two conditions (or just one) are not met, imposing the standard risk model on the data will not yield reliable results. It seems more reasonable to assume that in traditional less developed economies the above

[21] That is, the equivalent of region II in the case of more than two landlords and tenants.

two conditions are satisfied. As we move to more modern agricultural economies it becomes harder to make such a claim.

Another possibility (in some cases more realistic) is that both the landlords and the tenants are risk averse. Even in this case, one should not ignore the issue of the stability of matching. Hence, the theory outlined in this paper remains valid. With risk averse landlords there are additional considerations that should be addressed. One interesting implication is that if the variance is high, unlike the case with risk neutral landlords, the landlord may be better off cultivating the land himself. This is due to the fact that the high variability erodes the expected surplus and there is no contract which can give to both parties at least their reservation utilities, i.e., there is no room for "trade." In Serfes (2000) we study a model with one risk averse landlord and one risk averse tenant and we address the very interesting question of how the variance affects the share as well as the decision of the landlord to hire a tenant or not. The model yields testable implications and provides an *alternative* empirical specification for evaluating the role of risk in contracts.

5. Concluding remarks

The message of this paper is that risk may be playing a major role in shaping the nature of contracts, which may not be captured well the way the empirical models are structured. Most of the empirical as well as the theoretical papers on this issue ignore the stage in the game where the principal-agent pairs are formed and treat the degree of risk aversion as an exogenous variable. This stage may be very crucial especially in cases where there are profound differences in the degree of risk aversion among the agents. In this case, an econometric test should not be based on a single regression equation but rather on a system of simultaneous equations which would regard both the share and the degree of risk aversion as endogenous variables. In future work, we intend to test the validity of the risk model under the light of the theory outlined in this article.

Appendix

Proof of Proposition 4. What we need to show is that the algorithm we described above produces no cycles at any step.[22] First of all, there are no cycles in step 1 since the opportunity cost of each landlord is clear (i.e., tenant n) and the highest bidder wins tenant $n-1$. Now let's show that the same is true at any step. Consider the following representative example. Fix any step and assume that there were no cycles at the previous steps. Suppose that landlord 1 is the tentative winner and among all the remaining landlords landlord 3 outbids landlord 1 and wins tentatively the tenant. This change in the tentative winners may alter the opportunity costs of some landlords.

[22] A cycle would the following situation. Suppose landlord ℓ is the proposed winner of a tenant. Then landlord k has an incentive to bid more and becomes the new tentative winner. But now landlord m has an incentive to win. Given that m wins landlord ℓ has an incentive to bid more and he becomes the tentative winner.

Before we proceed with the proof we need the following definition. A landlord ℓ is said to be *displaced* by landlord k when the following events take place. Landlord k is the tentative winner of tenant t. Given this, landlord ℓ's opportunity cost is tenant (say) $t+3$. However, landlord k is outbid by some other (than ℓ) landlord and landlord k's opportunity cost is now tenant $t+3$ while landlord ℓ's opportunity cost becomes lower (say tenant $t+5$).

Case 1. If landlords 1 and 3 have the same opportunity costs when each is a tentative winner then no other tenant is displaced, the bids of all the other (than 1 and 3) landlords do not change and consequently there is no cycle. Hence the possibility of a cycle emerges when the tentative winner looses and displaces some other tenant. Assume that this is the case and move to case 2.

Case 2. Some other landlord, say landlord 5, has been displaced by landlord 1 and now has an incentive to bid more than landlord 3 and hire the tenant. (In fact more than one landlord may have been displaced but landlord 5 has the highest willingness to pay among those landlords). Is it now possible that landlord 1 has an incentive to outbid landlord 5? If this is the case we have a cycle. But if landlord 1 has an incentive to bid more than landlord 5 when landlord 5 is the tentative winner then he should have had the same incentive when landlord 3 was the tentative winner and landlord 5 outbid him. This is the case because when landlord 3 looses to landlord 5 he does not displace landlord 1 (this was case 1). However, when landlord 3 was the tentative winner landlord 5 outbid landlord 1. Hence, landlord 1 cannot outbid landlord 5 when the latter is the tentative winner and the possibility of a cycle at any given step vanishes.

An equilibrium is stable because there is no pair who can block it. This can be easily seen since the least risk averse tenant goes to the landlord who values him the most. Every landlord would like to match with this tenant but nobody is willing to bid more. The tenant with the second lowest risk aversion is hired by the landlord who values him the most among the remaining landlords and so on. The resulting matching therefore is stable. ■

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