Buying frenzies in durable-goods markets

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Abstract: We explain why a durable-goods monopolist would like to create a shortage during the launch phase of a new product. We argue that this incentive arises from the presence of a second-hand market and uncertainty about consumers' willingness to pay for the good. Consumers are heterogeneous in their valuations. Some consumers are initially uninformed about their valuations and learn about them over time while others are informed through their lifetimes. Given demand uncertainty, first period sales may result in misallocation and lead to active trading on secondary market after the uncertainty is resolved. We characterize conditions under which the monopolist would like to restrict sales and generate a buying frenzy. We show how the monopolist may benefit from an active second-hand market.

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1 Introduction

Introductions of new durable goods are often featured by serious shortage coupled with active trading on second-hand markets. Examples include video games, game consoles, iPad, iPhone and luxury cars. The repeated occurrence of shortage in new product markets suggests that firms may intentionally limit supplies to induce buying frenzies. Why does a firm ration consumers to the secondary market when it could have served them and made more sales? If the firm benefits from scarcity strategies, what is the mechanism behind it? How does the existence of second-hand market affect the firm's profit? These are the questions addressed in this paper.

The internet revolution has substantially enhanced active trading on second-hand markets when buying frenzies occur.[1] When iPad 2 was launched, Apple stores across the U.S. sold out of the tablet while the price of it spiked on eBay.[2] Similar phenomenon was documented for other electronics including Wii, PlayStation 2. Despite the important role played by the second-hand market, it is ignored by the existing literature aiming to explain firms' scarcity strategies. In fact, the predominant theories are not robust against resale. To the best of our knowledge, our paper is the first one to study durable-goods producers' incentive to induce buying frenzies while taking into account active trading on the second-hand market. Contrary to the existing literature, we argue that the existence of a second-hand market can be one of the driving forces for buying frenzies.

In our framework, consumers are heterogeneous in their valuations for the product. Some consumers are initially informed about their valuations while others learn about their valuations over time. When the seller sells to both informed and informed consumers, the product may end up with those who turn out to have low valuations. Hence, reallocation of the good among consumers takes place through the second-hand market when the uncertainty about consumers' valuations is resolved. Trading on the second-hand market will generate additional surplus. This surplus can be captured by the seller *ex ante* because consumers

[1] See, for example, Rapson and Schiraldi, 2011, for an empirical analysis of the internet impact on the trade volume of used cars on the second-hand market.

[2] "iPad 2 Prices Are Spiking on eBay", the Atalantic Wire, March 14, 2011.

are forward-looking, and the price they are willing to pay incorporates the product's resale value. As a consequence, when selling to both informed and uninformed consumers, the monopolist faces the trade-off between more sales today and a lower profit margin. Specifically, because the product's resale value is negatively related to the stock of the good in the second-hand market, selling more units today will result in a lower equilibrium price of the product. Therefore, the monopoly may find it optimal to create a shortage and ration consumers to the second period. Among consumers rationed to the second period, informed ones are strictly worse off because they prefer to consume the good in the first period but cannot do so. In contrast, uniformed consumers receive the same utilities no matter if they buy the good in period one or delay consumption.

We find it is profitable for the monopolist to restrict sales in the early period when 1) there is a large number of uninformed consumers, 2) few consumers have the lowest valuation, 3) the average valuation is low, or 4) the product's marginal cost is high.

Our paper is most closely related to DeGraba (1995). DeGraba argues when consumers learn their valuations over time, a monopolist can better extract consumer surplus by committing to a fixed output short of demand. The monopolist prefers to sell the good when consumers are uninformed. This is because it can extract the entire surplus when consumers have the same expected valuations, but has to give up some surplus when consumers become informed and end up with different valuations. When output is short of demand, consumers risk losing the opportunity to buy the good if they strategically delay purchases. As a consequence, consumers all rush to buy the good when they are uninformed. DeGraba does not allow consumers to resell the product when they become informed. In fact, the existence of an active second-hand market weakens DeGraba's theory. This is because the option of purchasing the good from the second-hand market reduces the risk borne by consumers when they delay consumption. Hence, it is difficult, if possible, for the monopolist to induce buying frenzies.

Several other papers including Denicolo and Garella (1999), Stock and Balachander (2005), Allen and Faulhaber (1991) have offered alternative theories for monopolist's scarcity strategies. Denicolo and Garella (1999) study a model without demand uncertainty. They argue that rationing reduces the monopolist's incentive to lower future prices and can convince consumers to buy without strategic delay. This may allow the monopolist to increase his discounted profit. Stock and Balachander (2005) and Allen and Faulhaber (1991) show that product scarcity can be used to signal a high quality of the product. None of these papers allows resale. In particular, in Denicolo and Garella, if consumers can resell, the arbitrage across periods will make the firm's rationing strategy less profitable.

In our model, the monopolist may prefer a smoothly functioning secondary market for a reason different from the existing literature (Swan (1980), Rust (1986), and Hendel and Lizzeri (1999)). When trade is driven by uncertainty in demand, secondary market can help the monopolist to extract surplus generated by reallocation of the good. In a similar context, Johnson (2011) studies the implications of uncertainty in demand and the presence of transaction costs on monopoly profit and its choice of durability.

In a related paper, Courty (2003b) studies a monopolist's selling strategy in ticket markets when there is demand uncertainty. In Courty, the monopolist sells either in early market when consumers are uninformed about their valuations for tickets or in a late market where their valuations are revealed. Despite the similar features shared with Courty, we have different findings. Courty found allowing resale does not improve the monopolist's profit. In contrast, we show the monopolist can benefit from allowing resale when the marginal cost is high enough. The driving force for the difference is because we focus on durable goods while Courty studies goods that can only be consumed once. One implication is that a monopolist is more likely to favor second-hand market when the product is more durable. This may explain why tickets sellers often take aggressive actions to kill off second-hand market[3], whereas durable goods sellers are more tolerant of second-hand market.

Finally, our paper is related to the literature on intertemporal pricing. Previous work has studied how the monopolist can use advance-purchase discount (Nocke et al. 2011, Dana 1998) or refund (Courty and Li 2000) to price discriminate between consumers when the uncertainty of consumers' valuations is resolved over time. Our paper differs from the previous work by allowing consumers to resell, and in particular, we

[3] Ticket promoters and agencies often attempt to exclude brokers from the secondary market. For example they restrict the number of tickets a single buyer can purchase at the box office or indicate in small print that tickets are nontransferable revocable licenses. Refer to Courty (2003a) for a detailed discussion on ticket resale.

focus on how the option of consumer resale affects the monopolist's optimal selling strategy.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium and shows the conditions under which buying frenzies occur. Section 4 discusses whether it is in the monopolist's interest to kill off the second-hand market. Section 5 discusses some extensions and shows how our model may explain one puzzling observation[4] in buying frenzies: the equilibrium price on the second-hand market is (sometimes) higher than the primary market. Section 6 concludes.

2 The model

In this section, we present a simple two-period model to illustrate the main idea. Section 5 discusses how the main results extend to a three-period model.

2.1 Players

A risk-neutral monopolist sells an indivisible durable good in two trading periods. There is a continuum of consumers who live for two periods. The mass of consumers is normalized to one. Consumers enter the market in period one, each buying, at most, one unit of the durable good in his life time. Consumers differ in their valuations for the product. A consumer is indexed by his valuation θ, with $\theta \in [0, \overline{\theta}]$. We interpret a consumer's valuation as his taste θ multiplied by the product's quality which is commonly known and normalized to one. The parameter θ is distributed according to cumulative distribution function $F(\theta)$ and density function $f(\theta)$. We assume $F(\theta)$ is strictly increasing and continuous on the interval $[0, \overline{\theta}]$. The distribution of θ is common knowledge. Consumers' valuations for the product are independent and identically distributed. If a consumer with valuation θ buys the product at price p in period t, $t = 1, 2$, his utility in period t is $\theta - p$. For expositional simplicity, we assume the monopolist and consumers do not discount.[5]

[4] During the Wii shortage, consumers traded actively on ebay and the average price paid for Wii has been around 150% to 180% of the retail price. (We want a Wii (Still)!, New York Times, 2007.) Sony's Playstation 2 was considered one of the hottest consumer electronics products available in 2000 (Retailing Today 2000). The launch of the product was characterized by shortage and it was being traded for up to 300% of its price in auctions.

[5] Our main results hold as long as the discount factor is not too low. When the discount factor is very low, consumers do not incorporate the product's resale value into their first period willingness to pay. This will destroy

There are β, $\beta \in [0, 1]$, fraction of informed consumers and $1 - \beta$ fraction of uninformed consumers. An informed consumer knows his valuation for the product through out his lifetime. In contrast, an uninformed consumer is ignorant about his valuation in period one, but learns about it in period two. We assume an uninformed consumer learns his valuation for the product regardless of whether he has purchased it in period one. This assumption is reasonable in a number of situations. For example, a gamer can learn how much he likes a game console by playing it at his friend's house or the game store; a car buyer will learn his valuation for a new car by test drive. We will discuss in Section 5 what happens if uninformed consumers learn their valuations only through consumption in period one. At the beginning of period 2, a second-hand market is opened. The good does not depreciate and consumers can resell the it at zero transaction cost.

The monopolist wishes to maximize the total profit from the two periods. The marginal cost of the product is assumed to be constant at c, with $0 < c \leq E(\theta) + \overline{\theta}$ [6]. When demand exceeds supply, consumers who wish to buy will obtain the good with equal probabilities.[7]

2.2 Timing

- Period 1. The monopolist decides first period price p_1 and supply q_1. Consumers decide whether to buy the good after observing (p_1, q_1).[8]

- Period 2. The monopolist decides price p_2 and output q_2. At the same time, a second-hand market is opened. Consumers observe (p_2, q_2) and the price on the second-hand market. Consumers who have owned the product decide whether to resell it, and consumers who haven't bought the product decide whether to buy the product and from whom to buy.

the monopolist's incentive to ration consumers in the first period.

[6] When $c > E(\theta) + \overline{\theta}$, uninformed consumers will never buy the product in period one. The product's highest possible resale value is $\overline{\theta}$. Hence, an uninformed consumer's maximum willingness to pay in period one is $E(\theta) + \overline{\theta}$.

[7] We could allow for more sophisticated forms of rationing but they would not change the main results as long as rationing is not fully efficient.

[8] The observability of the supply in period one reflects situations in which customers have some information about the availability of the good. For example, customers may know there is an import quota on a particular good, or they know that output will be limited because the seller announces a shortage of a necessary input such as computer chips. The observability of q_1 allows consumers to correctly perceive their true probabilities of getting the good.

3 The Equilibrium

This section characterizes the equilibrium and highlights the conditions under which the monopolist induces a buying frenzy. We say a buying frenzy occurs when the monopolist intentionally undersupplies the product to create a shortage. To illustrate the trade-off facing the monopolist, we first present a basic model when consumers are uninformed, and we show there is a continuum of equilibria with buying frenzies. In a buying frenzy, uninformed consumers are indifferent between receiving the product or being rationed out.

Building on the basic model, subsection 3.2 analyzes a richer and more realistic model with both informed and uninformed consumers. Unlike uninformed consumers, informed consumers lose out when rationed out in a buying frenzy because they strictly prefer to buy the product in the early period.

3.1 Basic Model

Assume consumers are all uninformed about their valuations in period one, i.e. $\beta = 0$. In the second period, given q_1, the monopolist and consumers make decisions to maximize their payoffs, respectively. Because consumers become informed about their valuations and can trade with each other, it is never optimal for the monopolist to ration consumers. Market clear condition requires the secondary market price equal p_2, the price charged by the monopolist. Second-hand market will allocate the product to consumers who value the product the most. Hence, marginal consumer's willingness to pay, $\widehat{\theta}$, satisfies $\int_{\widehat{\theta}}^{\overline{\theta}} f(\theta)d\theta = q_1 + q_2$, where the product is sold by the monopolist and consumers who own the product at the beginning of period two and wish to sell in secondary market.

The monopolist chooses p_2 and q_2 to maximize his second-period profit

$$\pi_2(p_2, q_2) = (p_2 - c)q_2$$

subject to the market-clear condition

$$F(\widehat{\theta}) = 1 - q_1 - q_2$$

and

$$q_2 \geq 0.$$

Substituting $\widehat{\theta} = p_2$, the interior solutions for p_2 and q_2 are determined by

$$p_2 - c = \frac{1 - F(p_2) - q_1}{f(p_2)} \tag{1}$$

and

$$q_2 = 1 - q_1 - F(p_2). \tag{2}$$

Assumption 1 $\dfrac{f^{'}(\theta)}{[f(\theta)]^2} > -2.$

Assumption 1 ensures the monopolist's second period profit is concave in q_2. Equations (1) and (2) imply p_2 is decreasing in q_1. This is because the more good the monopolist sells in period one, the more intense competition he will have to face against resellers, and, consequently, the lower the price in period two. We summarize the monopolist's output decision q_2 by the following lemma.

Lemma 1 *The monopolist will not sell the good in the second period when $q_1 \geq 1 - F(c)$ and will sell q_2 units otherwise, where q_2 is determined by $F^{-1}(1 - q_1 - q_2) - q_2 F^{-1'}(1 - q_1 - q_2) = c$.*

The monopolist will stop producing the product in period two if q_1 is at least $1 - F(c)$. The reallocation of q_1 units through the second-hand market will drive the marginal consumer's maximum willingness to pay to be, at most, the marginal cost. As a result, the monopolist will make a loss by producing more of the product. According to the solution of q_2, the monopolist's second-period profit is summarized by the next lemma.

Lemma 2 *The monopolist's second profit is zero when $q_1 \geq 1 - F(c)$ and $(p_2 - c)^2 f(p_2)$ otherwise, with p_2 determined by equation (1).*

When q_1 is less than $1 - F(c)$, the second-period price is higher than the marginal cost and the monopolist will earn a positive profit $(p_2 - c)q_2$, where p_2 and q_2 are determined by equations (1) and (2). Combining equations (1) and (2), we can write q_2 as $(p_2 - c)f(p_2)$. Accordingly, the monopolist's second period profit is $(p_2 - c)^2 f(p_2)$.

We are now ready to solve for the monopolist's problem in the first period. Consider an uninformed consumer's purchase decision in period one. If he buys the good immediately, he enjoys flow utility $E(\theta)-p_1$. In the next period, the consumer will keep the product if his valuation turns out to be greater than the resale price p_2; otherwise, he will sell the product and earns p_2. The consumer's expected payoff from purchasing the good in period one is therefore

$$E(\theta) - p_1 + [\Pr(\theta > p_2)E(\theta|\theta > p_2) + \Pr(\theta \leq p_2)p_2].$$

Alternatively, the consumer can delay consumption until his valuation is revealed in period two. Because the consumer will buy the good only when his valuation is greater than p_2, his expected payoff from waiting is $\Pr(\theta > p_2)[E(\theta|\theta > p_2) - p_2]$. As a result, the consumer will buy the product in period one if and only if $p_1 \leq E(\theta) + p_2$. Essentially, the second-hand market provides an insurance to uninformed consumers. When the good's resale value becomes higher, a consumer will bear a smaller loss if he turns out to have a low valuation. Hence, the consumer is willing to pay more for the good up front.

It is worthwhile to emphasize that when a consumer decides whether to purchase the good in period one, he needs to correctly form the expectation of the price in the second period, based on the monopolist's announcement of q_1. Specifically, consumers expect p_2 to be $F^{-1}(1-q_1)$ when the announced q_1 is at least $1-F(c)$. And, they expect p_2 to be determined by $p_2 - c = \dfrac{1-F(p_2)-q_1}{f(p_2)}$ when the announced q_1 is smaller than $1-F(c)$. Because the monopolist's profit function is continuous in q_1 but has a kink at $1-F(c)$, we cannot differentiate it at $q_1 = 1-F(c)$. Hence, we divide the monopolist's first period problem into two regimes, regime A and B. Regime A is the range $q_1 \in [0, 1-F(c))$ and regime B is the range $q_1 \in [1-F(c), 1]$. Let q_1^i, $i = A, B$, denote the optimum in regime i. The monopolist will choose the first period output $q_1^* \in \{q_1^A, q_1^B\}$ that results in the highest profit.

The monopolist's optimization problem in regime A is to choose q_1 to maximize

$$\pi_A(q_1) = (E(\theta) + p_2 - c)q_1 + (p_2 - c)^2 f(p_2) \tag{3}$$

where p_2 is a function of q_1 and is implicitly determined by

$$p_2 - c = \frac{1 - q_1 - F(p_2)}{f(p_2)}, \tag{4}$$

subject to

$$0 \le q_1 < 1 - F(c).$$

Assumption 2 $\dfrac{3f'(p_2) + (p_2 - c)f''(p_2)}{(2f(p_2) + (p_2 - c)f'(p_2))^2} > -1, \ \forall p_2 \ge c.$

Assumption 2 ensures the monopolist's profit function in regime A is concave in q_1. In regime B, the monopolist only produces in period one. Consequently, the product's resale value is $F^{-1}(1 - q_1)$, and consumers' maximum willingness to pay in period one becomes $E(\theta) + F^{-1}(1 - q_1)$. The monopolist's optimization problem in regime B is to choose q_1 to maximize

$$\pi_B(q_1) = (E(\theta) + F^{-1}(1 - q_1) - c)q_1 \tag{5}$$

subject to

$$1 - F(c) \le q_1 \le 1.$$

Assumption 1 ensures the monopolist's profit function in regime B is concave in q_1.

Next, we characterize conditions under which a buying frenzy occurs in period one.

Proposition 1 *There exists a continuum of equilibria with buying frenzies if and only if* $E(\theta) - \dfrac{1}{f(0)} < c$. *In a buying frenzy, the monopolist charges* $p_1 = E(\theta) + p_2$ *and the demand exceeds the supply.*

When choosing the first-period output, the monopolist faces the trade-off between more sales in period one and lower profit margins. By selling more of the product today, the monopolist will increase the stock in the second-hand market and hence reduce the product's resale value. As a consequence, a forward looking consumer is willing to pay less up front expecting a lower resale value. To better understand proposition 1, imagine the monopolist sells to all uninformed consumers in period one. Because no one will buy the product in the second-hand market, the market-clear condition implies the product's resale value is zero. Marginal benefit from undercutting the first-period output by one unit is $\dfrac{1}{f(0)}$, which is the marginal increase in the product's resale value. Marginal cost of it is $E(\theta) - c$, the forgone profit per unit in the first period. When the condition $E(\theta) - \dfrac{1}{f(0)} < c$ holds, the marginal benefit of undercutting the first-period output outweighs the marginal cost.

Because the monopolist charges consumers' maximum willingness to pay in period one, consumers are just indifferent between purchasing the good immediately and waiting. However, in equilibrium, demand must be at least the supply. To see this, suppose the monopolist's optimal profit π^* is achieved at $q_1^* < 1$. If the demand is less than q_1^*, the monopolist can achieve a profit arbitrarily close to π^* by undercutting price slightly below $E(\theta) + p_2$ and still managing to sell q_1^* units. While there is a continuum of equilibria with excess demand, we want to point out there does exist an equilibrium in which the market just clears.

Proposition 1 implies the monopolist is more likely to restrict sales in period one when 1) there are very few consumers with the lowest valuation, 2) the average valuation is low, or 3) the product's marginal cost is high. To see the first implication, imagine the distribution of θ has a thin tail at 0, restricting the first-period sales will boost the product's resale value substantially because most consumers will turn out to have high valuations. Hence, undercutting output in period one will significantly increase consumers' first period willingness to pay. Implication 2) and 3) are straightforward. When the product's resale value is zero, consumers' first-period maximum willingness to pay is $E(\theta)$. A lower average valuation or a larger marginal cost will result in a lower profit margin in the first period. As a result, it is less costly for the monopolist to restrict sales in period one.

Two possible scenarios may occur when the monopolist induces a buying frenzy. The monopolist may sell to a fraction of consumers in period one and does not sell in period two. Or, he may sell in both periods. According to lemma 1, the monopolist should keep producing in period two if and only if he sells less than $1 - F(c)$ units in period one.

Corollary 1 *When a buying frenzy occurs in period one, the monopolist sells in period two if and only if* $E(\theta) < \dfrac{1 - F(c)}{2f(c)}$.

The monopolist's second period output depends on consumers' average valuation. When the average valuation is low, the monopolist will sell to very few consumers in period one. The intuition is best understood by supposing that the average valuation is far below marginal cost. In order to make a positive profit in period one, the monopolist has to restrict sales aggressively. Doing so can boost consumers' first-period willingness to pay high enough to cover the marginal cost of production. As a result of the limited first-period sale, the product's resale price is maintained above the marginal cost. This makes it profitable for the monopolist to keep producing in period two. In contrast, if the average valuation is already high enough to ensure a high profit margin in period one, it is too costly for the monopolist to cut sales in period one. So, the product's resale value falls below the marginal cost which prevents the monopolist from producing in period two.

The trade-off facing the monopolist is well captured by the basic model. In the next subsection, we discuss the more realistic model with both informed and uninformed consumers. We show the monopolist still has the incentive to create a shortage, and in a buying frenzy informed consumers are strictly worse off when rationed to the second period.

3.2 Heterogenous Consumers

Because consumers are all informed in the second period, the monopolist faces the same problem in the second period as in the basic model. Now, we consider the monopolist's first period problem. Uninformed consumers' first period purchase decision has been analyzed in the basic model. Similar as uninformed consumers, an informed consumer with valuation θ gets flow utility $\theta - p_1$ from consuming the product in period one. In the next period, he keeps the product if the resale price is at most θ and resells the product otherwise. Hence, the consumer's payoff from purchasing the product in period one is

$$\theta - p_1 + [I(\theta \geq p_2)\theta + (1 - I(\theta \geq p_2))p_2]$$

, where $I(.)$ is the indicator function. If the consumer postpones the purchase to period two, he will buy the product if and only if his valuation is larger than p_2. Consequently, consumer θ should purchase the product in period one if and only if

$$\theta - p_1 + [I(\theta \geq p_2)\theta + I(\theta < p_2)p_2] \geq I(\theta \geq p_2)(\theta - p_2)$$

, and hence his maximum willingness to pay in period one is $\theta + p_2$.

If the monopolist charges p_1 in period one, demand from informed consumers is $D_I(p_1) = [1 - F(p_1 - p_2)]\,\beta$. Demand from uninformed consumers is $1 - \beta$ if $p_1 \leq E(\theta) + p_2$, and zero, otherwise. Accordingly, the monopolist's first period problem is to solve the following program

$$\max_{p_1, q_1} (p_1 - c)q_1 + \pi(p_2) \tag{6}$$

subject to

$$0 \leq q_1 \leq \beta(1 - F(p_1 - p_2)) + (1 - \beta)I(p_1 \leq E(\theta) + p_2) \tag{7}$$

where $\pi(p_2)$ is characterized in lemma 2.

The monopolist should choose between selling to informed consumers only by charging a price higher than uninformed consumers' maximum willingness to pay, and selling to both informed and uninformed consumers at a lower price. Specifically, if $p_1 > E(\theta) + p_2$, only informed consumers whose valuations are at least $p_1 - p_2$ will buy the product. When p_1 is at most $E(\theta) + p_2$, the cohort of uninformed consumers will buy the product along with informed consumers with valuations greater than p_1.

Lemma 3 *Buying frenzies will not occur if the monopolist charges a first period price other than uninformed consumers' maximum willingness to pay $E(\theta) + p_2$.*

To see lemma 3, suppose the monopolist sells to a fraction of informed consumers who wish to buy the good at $p_1 > E(\theta) + p_2$. The monopolist can make more profit by selling the same units at a higher price $p_1^{'}$ where the demand at $p_1^{'}$ equals supply. Lemma 2 shows the monopolist's second period profit only depends on his first period sales q_1. Hence, this alternative selling strategy won't affect the monopolist's second period profit but will yield more profit in the first period. Following the same logic, the monopolist will not ration consumers at $p_1 < E(\theta) + p_2$. Lemma 3 implies the monopolist may have an incentive to ration consumers only when he charges a price equal uninformed consumers' maximum willingness to pay.

Proposition 2 *At $p_1 = E(\theta) + p_2$, for each $c > E(\theta) - \dfrac{1}{f(0)}$, there exists $\beta^* \in \left(0, min\left\{\dfrac{F(c)}{F(E(\theta))}, 1\right\}\right)$, such that the monopolist induces a buying frenzy for $\beta < \beta^*$. In the buying frenzy, informed consumers strictly prefer to buy in the first period while uninformed consumers are just indifferent between buying in* *period one and waiting. When* $0 < c < E(\theta) - \dfrac{1}{f(0)}$, $\forall\ \beta \in [0,1]$, *the monopolist sells to all consumers who wish to buy.*

When the monopolist targets the cohort of uninformed consumers, he faces the similar trade-off in the basic model. However, different from the basic model, the product has a positive resale price even if the monopolist sells to every one who wishes to buy at the first period price. This is because the good may be allocated to uninformed consumers who turn out to have extremely low valuations. These consumers will find it profitable to sell the product to informed consumers who have higher valuations but do not get to buy the good in period one.

Lemma 3 has shown buying frenzies will not occur had the monopolist targeted informed consumers only. Hence, the scarcity strategy is more appealing when there are sufficiently many uninformed consumers. With a general distribution, it is difficult to solve the model explicitly. Nevertheless, we can provide an example with discrete valuations to show this point.

Example 1. Discrete valuations and 2 periods. Suppose there is a mass one of heterogenous consumers. Specifically, there are two types of consumers: h fraction of are high- type consumers with valuations are $\bar{\theta} > 0$, and $(1-h)$ are low-type consumers with valuation 0. To focus on the most interesting case, we assume $h \geq 0.5$ to guarantee a positive price on an active secondary market. Moreover, suppose that a fraction β of all consumers are informed and $(1-\beta)$ are uninformed.

Suppose the marginal cost is $c \in (0, \bar{\theta})$.So the monopolist may only sell to high-type consumers in the second period. In this environment, the monopolist can choose among three alternatives selling schemes in period one according to different ranges of the parameters.

EP. The monopolist charges uninformed consumers' maximum willingness to pay and sell to all consumers who wish to buy. In this case the optimal price is equal to $\bar{\theta}h$, which is the expected value in period 1 (a consumer can purchase the product for free in the second-hand market if he decides to wait). In equilibrium, there is no active second-hand market and no new units are sold in the second period. The monopolist profit is $(\bar{\theta}h - c)(1 - \beta + \beta h)$.

HP. The monopolist excludes uninformed consumers and only sell to informed consumers with high valuations. Specifically, he will sell βh units in period one at price $2\bar{\theta}$ and $(1-\beta)h$ units in period two at price $\bar{\theta}$ to the consumers who become informed about their valuations for the good. The monopolist profit will be $(2\bar{\theta}-c)\beta h+(\bar{\theta}-c)(1-\beta)h$.

RP. Finally, the monopolist may charge uninformed consumers' maximum willingness to pay and ration consumers at this price. In period one, the monopolist will always sell a quantity equal to the fraction of high type consumers, i.e. $q_1 = h$. To see this, note that the monopolist will not sell a quantity between h and $h+(1-\beta)(1-h)$ because the product's resale price drop to zero if the monopolist sells more than h.[9] Selling less than h is not optimal. Suppose $q_1 < h$, the monopolist will find it optimal to sell $h-q_1$ units to the high type in the second period at price h. But then the monopolist can do better by selling h units in the first period. Doing so will not affect the resale price and will allow the monopolist to extract more surplus in the first period because consumers are willing to pay more when they consume the good in both periods. At price $p_1 = h\bar{\theta}+\bar{\theta}$, where $h\bar{\theta}$ is consumers' expected valuation and $\bar{\theta}$ is the product's resale value, $\dfrac{\beta h^2}{\beta h+(1-\beta)}$ units are bought by the high types of informed consumers ($\beta h - \dfrac{\beta h^2}{\beta h+(1-\beta)}$ informed consumers are rationed) and $\dfrac{h(1-\beta)}{\beta h+(1-\beta)}$ units are bought by the uninformed consumers ($1-\beta-\dfrac{h(1-\beta)}{\beta h+(1-\beta)}$uninformed consumers are rationed). The optimal profit is then $(h\bar{\theta}+\bar{\theta}-c)h$.

A simple comparison shows that RP is always better than EP and RP>HP if and only if $\beta < h$. HP is preferable to RP (and EP) if the mass of informed consumers is higher than the high valuation types.[10]

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[9] Such strategy will be strictly dominated by EP

[10] More generally if $\underline{\theta} > 0$, RP>EP if $\underline{\theta} < \frac{c(1-h)}{1+(1-h)^2}$ or $\underline{\theta} \geq \frac{c(1-h)}{1+(1-h)^2}$ and $\bar{\theta} > \frac{\underline{\theta}(1+(1-h)^2)-c(1-h)}{h^2}$ or $\underline{\theta} \geq \frac{c(1-h)}{1+(1-h)^2}$, $\bar{\theta} \leq \frac{\underline{\theta}(1+(1-h)^2)-c(1-h)}{h^2}$ and $\beta > \frac{(\bar{\theta}h+\underline{\theta}(1-h)-c)(1-h)+\underline{\theta}-\bar{\theta}h}{(\bar{\theta}h+\underline{\theta}(1-h)-c)(1-h)+\underline{\theta}(1-h)}$

Table 1 shows a numerical example with $\beta = 0.6$, $h = 0.7$, $c = 0.3$, $\bar{\theta} = 0.6$.

	RP	**EP**	**HP**
q_1	0.7	0.82	0.42
p_1	1.02	0.42	1.2
q_2	0	0	0.28
p_2	0.6	0	0.6
quantity traded on the secondary market	0.102	0	0
rationed informed consumers	0.061	0	0
rationed uninformed consumers	0.059	0	0
profit	0.504	0.098	0.462

Table 1

4 Banning Secondary Market

If goods are perfect substitutes, the monopolist might prefer to avoid the competition with used units by closing the second-hand market. Our framework looks at a different prospective by introducing uncertainty about consumers' valuations of the product. We show how a monopolist might benefit from an active second-hand market because it improves the allocation of the good among consumers with different valuations once the uncertainty is resolved. During the launch of the good, the uncertainty about consumers' valuations is likely to generate a misallocation of the good. The possibility of trading the good on the second-hand market generates a larger consumer surplus that will be captured by the monopolist at the time the product is launched.[11] For clarity of exposition and to focus on the main aspect of this section, we restrict our analysis to the basic case with $\beta = 0$.

Without the second-hand market, the monopolist faces a standard static problem in the second period because all consumers are informed about their valuations. Given that q_1 units of the product have been sold, he chooses p_2 to maximize $\pi_2 = (p_2 - c)(1 - F(p_2))(1 - q_1)$. The optimal p_2 is determined by $p_2 - c = \dfrac{1 - F(p_2)}{f(p_2)}$. In period one, since the product does not have a resale value, a consumer's expected payoff from buying it immediately is $E(\theta) - p_1 + E(\theta)$; his expected payoff from waiting until period two is $(1 - F(p_2))\,(E(\theta|\theta > p_2) - p_2)$. Hence, the consumer will buy the product in period one if and only if $p_1 \leq E(\theta) + [F(p_2)E(\theta|\theta \leq p_2) + (1 - F(p_2))\,p_2]$. The monopolist's first-period problem is to choose q_1 and

[11] This mechanism will generate extra surplus even if we allow for a depreciation of the good in the second period.

p_2 to maximize

$$\pi(q_1, p_2) = \{E(\theta) + [F(p_2)E(\theta|\theta \leq p_2) + (1 - F(p_2))\, p_2] - c\}\, q_1 + (p_2 - c)(1 - F(p_2))(1 - q_1)$$

subject to

$$p_2 - c = \frac{1 - F(p_2)}{f(p_2)}. \tag{8}$$

Without the second-hand market, consumers' first-period willingness to pay does not depend on the first-period sales. Consequently, $\pi(q_1, p_2)$ is a weighted average of the profit margins in two periods, with the weights q_1 and $1 - q_1$ respectively. The monopolist will sell to all consumers in period one if and only if the profit margin in period one is greater than period two. Specifically, the monopolist's solution is

$$q_1 = \begin{cases} 0 & \text{if } c > \dfrac{E(\theta) + F(p_2)E(\theta|\theta \leq p_2)}{F(p_2)} \\ [0,1] & \text{if } c = \dfrac{E(\theta) + F(p_2)E(\theta|\theta \leq p_2)}{F(p_2)} \\ 1 & \text{if } c < \dfrac{E(\theta) + F(p_2)E(\theta|\theta \leq p_2)}{F(p_2)} \end{cases},$$

where p_2 is determined by equation (8).

When $c = \dfrac{E(\theta) + F(p_2)E(\theta|\theta \leq p_2)}{F(p_2)}$, the monopolist is just indifferent between selling in period one or two. In other words, selling in both periods will not yield the monopolist a profit higher than selling in a single period. In sharp contrast, when the monopolist allows consumers to trade on the second-hand market, he may strictly prefers to sell in both periods (see corollary 1).

Proposition 3 *When $\max\{\overline{\theta}, 2E(\theta)\} \leq c < E(\theta) + \overline{\theta}$, the monopolist prefers to have the second-hand market.*

Clearly, when $\overline{\theta} \leq c$, the monopolist will make a zero profit in the second period regardless of whether there is a second-hand market. Hence, the monopolist prefers the second-hand market only when it can make more profit in period one by allowing resale. Consider the scenario without a second-hand market. The surplus per unit from selling in period one is $2E(\theta) - c$. If c is larger than $2E(\theta)$, the surplus is negative and hence the monopolist will make a loss by selling in period one. However, it may be profitable for the monopolist to sell in this case when there is a second-hand market. This is because trading on the second-hand market improves allocation efficiency, and hence generates a larger surplus in period one. For example, if the monopolist sells to only one consumer in period one, the product will be traded to the consumer with the highest valuation in period two. Hence, the social surplus generated in period one is $E(\theta) + \overline{\theta} - c$.

5 Discussion

Learning from experience In the main model, an uninformed consumer's valuation is revealed to him in the second period no matter whether he has purchased the product in the first period. An alternative learning rule is that a consumer learns his valuation only through consumption in the first period. In this case, if an uninformed consumer does not purchase the good in the first period, his maximum willingness to pay in the second period is still the expected valuation. Hence, learning from experience puts a constraint on the product's resale value. However, as long as the product's resale value is positive, the trade-off between more sales in the first period and a lower profit margin in the first period still exists. We can still find the parameter values that make it more profitable for the monopolist to sell below the quantity demanded.

Multiple-period model The main idea can be easily extended to an overlapping generation model in which a cohort of old informed consumers are mixed with a cohort of new uninformed consumers in each period. The existence of both informed and uninformed consumers may induce the monopolist to charge a price different from the price in the second-hand market.

Intuitively, when the size of uninformed new consumers is relatively larger than the informed old consumers, it is more profitable for the monopolist to target the uninformed cohort (the argument is similar to the previous discussion about heterogeneous consumers). By targeting the new uninformed consumers, the price on the primary market will be, at most, the sum of the expected valuation and the product's future resale value. For the same reason presented in the main model, the monopolist may want to ration consumers in order to maintain a high resale value of the product. Given the proportional rationing assumption, informed and uninformed consumers are equally likely to be rationed to the second-hand market. The willingness to pay of the marginal informed consumer who has been rationed by the monopolist and is just indifferent between whether or not to buy the good on the second-hand market will be determined by the price on second-hand market. Everything else equal, the lower is the quantity traded on the second-hand market, the higher is the equilibrium price on the second-hand market. If consumers do not buy and resell the product in the same period, the product's resale price could be higher than its primary price when there are very few units available on the second-hand market. This prediction fits the observation that during a buying frenzy, a product's resale value is sometimes higher than its primary price. We modify the example of discrete valuations with two periods to show this point.

Example 2. Discrete valuations and 3 periods. In the 3-period model, we assume that in the second period there is a new cohort of mass-one heterogenous consumers entering the market who has the same composition as the cohort in the period one. As in example 1 in a 3-period model, there are three equivalent possible optimal monopoly strategies. Under EP the monopoly will sell only in the first two periods and charge $p_1 = 2h\bar{\theta} + \bar{\theta}$ and $p_2 = h\bar{\theta} + \bar{\theta}$, and sell to all consumers who wish to buy, i.e. the uninformed consumers and the high types informed consumers. Under HP, the monopolist sells only to the informed high types at $p_1 = 3\bar{\theta}$, $p_2 = 2\bar{\theta}$ and $p_3 = \bar{\theta}$. Finally under RP, the monopolist will sell only in the first two periods at $p_1 = h\bar{\theta} + 2\bar{\theta}$ and $p_2 = h\bar{\theta} + \bar{\theta}$. The prices are determined as in example 1 except for taking into account of an extra period in determining p_1. The monopolist will sell overall $2h$ units and specifically $q_1 = (1 - \beta) + \beta h$ and $q_2 = 2h - q_1$ with proportional rationing in period 2 only. We focus the attention to the case where $\beta = 0.6$, $h = 0.7$, $c = 0.3$ *and* $\bar{\theta} = 0.6$. As discussed above under a RP strategy, a 3-period model generates an equilibrium price on the second-hand market which is higher than the primary-market equilibrium price. In fact in the second period, the demand for new units at the price 1.02 is equal to the informed high type consumers entering in the second period, i.e. 0.42, plus all the uninformed consumers, i.e. 0.4. Because the amount sold by the monopolist in the second period is 0.58, the proportional rationing will lead to 0.123 informed consumers, who are willing to pay up to 1.2 to get the good, rationed and 0.117 uninformed consumers, who are willing to pay up to 1.02, rationed. In the secondary market, there will be 0.12 units available from the low type who own the good. These consumers can extract all the surplus from the high type by charging a price equal to 1.2 on the secondary market. Table 2 summarizes the solution of the example.

	RP	**EP**	**HP**
q_1	0.82	0.82	0.42
p_1	1.62	0.84	1.8
q_2	0.58	0.7	0.7
p_2	1.02	0.42	1.2
resale price in period 2	1.2	0.42	1.2
rationed informed consumers in period 2	0.003	0	0
rationed uninformed consumers in period 2	0.117	0	0
q_3	0	0	0.28
p_3	0.6	0	0.6
profit	1.674	0.737	1.344

Table 2

6 Conclusion

This paper explains why a durable-goods monopolist would like to restrict supply and induce a buying frenzy in the presence of an active second-hand market and demand uncertainty. While the existing literature ignores the important role played by the second-hand market, we argue that the option of reselling the product on the second-hand market can be one of the driving forces for this firm's strategy. As is pointed out by the existing literature that the existence of second-hand market forces the monopolist to compete with resellers, we show the monopolist may still prefer to have a second-hand market. This is because trading on a second-hand market can improve allocation efficiency in the presence of demand uncertainty and hence increase the monopolist's profit.

Finally, we emphasize that our explanation does not exclude other explanations for product scarcity. In particular, the scarcity of fashion products can also be driven by consumers' need for exclusivity. Moreover, similar behavior could be explained in a context where firms can influence social learning among consumers by manipulating the launch sequence of a new product. It is noted that it can be profitable for a firm to restrict the access of a new product to consumers in order to induce a purchasing herd (Liu and Schiraldi 2011).

Appendix

Proof for lemma 1: First, we show $q_2 = 0, \forall\, q_1 \geq 1-F(c)$. Under assumption 1, marginal revenue function $MR(q_2) = F^{-1}(1-q_1-q_2)-q_2F^{-1'}(1-q_1-q_2)$ is decreasing and reaches the maximum $F^{-1}(1-q_1)$ at $q_2 = 0$.

Since $F^{-1}(1-q_1)$ is strictly decreasing in q_1, $F^{-1}(1-q_1) \leq F^{-1}(1-(1-F(c))) = c$ for $q_1 \geq 1-F(c)$. This implies the marginal revenue of q_2, $\forall\ q_2 > 0$, is always smaller than the marginal cost when $q_1 \geq 1-F(c)$. So, $q_2 = 0$, $\forall\ q_1 \geq 1-F(c)$.

Next, we show q_2 is determined by

$$F^{-1}(1-q_1-q_2) - q_2 F^{-1'}(1-q_1-q_2) = c \tag{9}$$

for $q_2 < 1-q_1$. When $q_1 < 1-F(c)$, marginal revenue of q_2 evaluated at $q_2 = 0$ is larger than c. Hence q_2 is either the interior solution determined by (9) or the corner solution $q_2 = 1-q_1$. Next, we rule out the corner solution. Equation (1) implies that $p_2 > c$ when $q_1 < 1-F(c)$. To see this, suppose $p_2 \leq c$. The left hand side of equation (1) is nonpositive. When $q_1 < 1-F(c)$ the right hand side of equation (1) is larger than $\dfrac{F(c)-F(p_2)}{f(p_2)} > 0$. A contradiction. Because $p_2 > c > 0$, $q_2 < 1-q_1$. Suppose $q_2 = 1-q_1$, by equation (2), $p_2 = 0$. A contradiction. Q.E.D.

Proof for proposition 1: Let q_1^* denote the monopolist's optimal output in period one. We first show $E(\theta) - \dfrac{1}{f(0)} < c$ is a sufficient condition for $q_1^* < 1$. By lemmas 1 and 2, when $q_1 \geq 1-F(c)$ (regime B), the monopolist earns zero profit in period two. The derivative of $\pi_B(q_1)$ with respect to q_1 is

$$\begin{aligned}\pi'_B(q_1) &= E(\theta) + F^{-1}(1-q_1) - c - F^{-1\prime}(1-q_1)q_1 \\ &= E(\theta) + F^{-1}(1-q_1) - c - \frac{q_1}{f\left(F^{-1}(1-q_1)\right)}.\end{aligned}$$

Evaluate $\pi'_B(q_1)$ at $q_1 = 1$, we have $\pi'_B(q_1)|_{q_1=1} = E(\theta) + F^{-1}(0) - c - \dfrac{1}{f\left(F^{-1}(0)\right)}$. Substitute $F^{-1}(0) = 0$, $\pi'_B(q_1)|_{q_1=1} = E(\theta) - c - \dfrac{1}{f\,(0)}$. Hence, when $E(\theta) - \dfrac{1}{f(0)} < c$, $\pi'_B(q_1)|_{q_1=1} < 0$, and the monopolist can improve his profit by undercutting the output below $q_1 = 1$.

Next, we show $E(\theta) - \dfrac{1}{f(0)} < c$ is a necessary condition for $q_1^* < 1$. First, we show the monopolist's total profit function is concave in q_1, for $q_1 \in [1-F(c), 1]$ (regime B). The second derivative of the profit function in regime B is

$$\begin{aligned}\pi''_B(q_1) &= -F^{-1'}(1-q_1) - F^{-1\prime}(1-q_1) + F^{-1''}(1-q_1)q_1 \\ &= \frac{-2}{f\left(F^{-1}(1-q_1)\right)} - \frac{q_1 f'\left(F^{-1}(1-q_1)\right)}{f^3\left(F^{-1}(1-q_1)\right)} \\ &= \frac{-1}{f\,(p_2)}\left[2 + \frac{q_1 f'\,(p_2)}{f^2\,(p_2)}\right].\end{aligned}$$

By assumption 1, $f^{'}(p_2) > -2f^2(p_2)$. So, $2 + \frac{q_1 f^{'}(p_2)}{f^2(p_2)} > 2(1-q_1) \geq 0$. Accordingly, $\pi_B^{''}(q_1) < 0$.

Then we show the monopolist's profit function is concave in q_1, for $q_1 \in [0, 1-F(c))$ (regime A). Take the derivative of $\pi_A(q_1)$ with respect to q_1 and substitute $\frac{\partial p_2}{\partial q_1} = -\frac{1}{2f(p_2)+(p_2-c)f'(p_2)}$, we have

$$\begin{aligned}
\pi_A^{'}(q_1) &= E(\theta) + p_2 - c + \left[q_1 + 2(p_2-c)f(p_2) + (p_2-c)^2 f'(p_2)\right]\frac{\partial p_2}{\partial q_1} && (10)\\
&= E(\theta) - \frac{q_1}{2f(p_2)+(p_2-c)f'(p_2)}\\
&= E(\theta) + q_1\frac{\partial p_2}{\partial q_1}. && (11)
\end{aligned}$$

The second derivative is therefore

$$\begin{aligned}
\pi_A^{''}(q_1) &= \frac{\partial p_2}{\partial q_1} + q_1\frac{\partial^2 p_2}{\partial q_1^2}\\
&= \frac{\partial p_2}{\partial q_1} + \frac{q_1(3f'(p_2)+(p_2-c)f''(p_2))}{(2f(p_2)+(p_2-c)f'(p_2))^2}\frac{\partial p_2}{\partial q_1}\\
&= \frac{\partial p_2}{\partial q_1}\{1 + q_1\frac{(3f'(p_2)+(p_2-c)f''(p_2))}{(2f(p_2)+(p_2-c)f'(p_2))^2}\}.
\end{aligned}$$

Assumption 1 and assumption 2 imply $\frac{\partial p_2}{\partial q_1} < 0$ and $1 + q_1\frac{(3f'(p_2)+(p_2-c)f''(p_2))}{(2f(p_2)+(p_2-c)f'(p_2))^2} > 0$, respectively. Hence, $\pi_A^{''}(q_1) < 0$.

Finally, we show the monopolist's total profit function is continuous and globally concave in q_1. Because $p_2 = c$ at $q_1 = 1-F(c)$. The monopolist's profit in regime A at $q_1 = 1-F(c)$ is $E(\theta)(1-F(c))$, which equals to the profit in regime B at $q_1 = 1-F(c)$. Since the monopolist's profit is concave in regimes A and B, it is globally concave if $\pi_A^{'}(q_1)|_{q_1=1-F(c)} \geq \pi_B^{'}(q_1)|_{q_1=1-F(c)}$. Lemma 1 has shown $p_2 = c$ when $q_1 = 1-F(c)$. Hence,

$$\pi_A^{'}(q_1)|_{q_1=1-F(c)} = E(\theta) - \frac{1-F(c)}{2f(c)}.$$

Similarly,

$$\begin{aligned}
\pi_B^{'}(q_1)|_{q_1=1-F(c)} &= E(\theta) + F^{-1}(F(c)) - c - \frac{1-F(c)}{f\left(F^{-1}(F(c))\right)}\\
&= E(\theta) - \frac{1-F(c)}{f(c)}.
\end{aligned}$$

Clearly, $\pi_A^{'}(q_1)|_{q_1=1-F(c)} \geq \pi_B^{'}(q_1)|_{q_1=1-F(c)}$. Because the profit function is globally concave in q_1, $E(\theta) - \frac{1}{f(0)} < c$ is a necessary condition for $q_1^* < 1$. Q.E.D.

Proof for corollary 1: According to lemma 1, the monopolist will sell in period two if and only if its optimal first period output $q_1^* < 1 - F(c)$. Proof of proposition 1 has shown the the monopolist's total profit function is globally concave in q_1. Consequently, $q_1^* < 1 - F(c)$ if and only if the derivative of the total profit function in regime A is negative when evaluated at $q_1 = 1 - F(c)$. Refer to the proof of proposition 1,

$$\pi_A^{'}(q_1)|_{q_1=1-F(c)} = E(\theta) - \frac{1 - F(c)}{2f(c)}.$$

When $E(\theta) < \dfrac{1 - F(c)}{2f(c)}$, the monopolist chooses $q_1^* < 1 - F(c)$ and sells in both periods. Otherwise, he only sells in period one. Q.E.D.

Proof for lemma 3: Let (p_1^*, q_1^*) denote the monopolist's solution for program (6). Suppose $p_1^* > E(\theta) + p_2$. Given p_1^*, the monopolist's first period demand is $\beta(1 - F(p_1^* - p_2))$. Suppose $q_1^* < \beta(1 - F(p_1^* - p_2))$. The monopolist can make more profit by charging $p_1^{'} > p_1^*$ such that $q_1^* = \beta(1 - F(p_1^{'} - p_2))$ and still sells q_1^* units. By lemma 2, $\pi(p_2)$ only depends on q_1. Therefore $(q_1^*, p_1^{'})$ yields the same second period profit as (q_1^*, p_1^*) but generates more first period profit than (p_1^*, q_1^*). A contradiction.

Suppose $p_1^* < E(\theta) + p_2$. Given p_1^*, the monopolist's first period demand is $\beta(1 - F(p_1^* - p_2)) + (1 - \beta)$. Suppose $q_1^* < \beta(1 - F(p_1^* - p_2)) + (1 - \beta)$. The monopolist can sell q_1^* units at a higher price $p_1^{'} = p_1^* + \varepsilon, \varepsilon > 0$ and $\varepsilon \to 0$. To see this, when ε is sufficiently close to 0, $p_1^{'} < E(\theta) + p_2$ and $q_1^* \leq \beta(1 - F(p_1^{'} - p_2)) + (1 - \beta)$. Similar as the argument in the case of $p_1^* > E(\theta) + p_2$, $(q_1^*, p_1^{'})$ is more profitable than (p_1^*, q_1^*). A contradiction. Q.E.D.

Proof for proposition 2: Step 1 shows for each $c > E(\theta) - \dfrac{1}{f(0)}$, the monopolist induces a buying frenzy for β small enough. Let $\widetilde{q}_1$ denote the optimal units the monopolist wishes to sell in period 1 at $p_1 = E(\theta) + p_2$, and $\widetilde{Q}_1$ denotes the first period demand facing the monopolist at this price. Demand $\widetilde{Q}_1 = (1 - \beta) + \beta(1 - F(E(\theta)))$, where $1 - \beta$ is the demand from uninformed consumers and $\beta(1 - F(E(\theta)))$ is the demand from informed consumers. For each $c > E(\theta) - \dfrac{1}{f(0)}$, $1 - F(c) < q_1$, provided that $\beta < min\left\{\dfrac{F(c)}{F(E(\theta))}, 1\right\}$. By lemmas 1 and 2, the monopolist produces nothing and earns zero profit in period 2 if he sells more than $1 - F(c)$ in period 1. The monopolist's problem in regime B is to choose q_1 to maximize objective function (5) subject to $1 - F(c) \leq q_1 \leq \widetilde{Q}_1$. Take the derivative of $\pi_B(q_1)$ and evaluate $\pi_B'(q_1)$ at

$q_1 = \widetilde{Q}_1$, we have

$$\begin{aligned} & E(\theta) - c + F^{-1}(1 - \widetilde{Q}_1) - \frac{\widetilde{Q}_1}{f(F^{-1}(1 - \widetilde{Q}_1))} \\ = \quad & E(\theta) - c + \left\{ F^{-1}(\beta F(E(\theta))) - \frac{1 - \beta F(E(\theta))}{f(F^{-1}(\beta F(E(\theta))))} \right\}. \end{aligned} \tag{12}$$

Define

$$G(\beta) \equiv F^{-1}(\beta F(E(\theta))) - \frac{1 - \beta F(E(\theta))}{f(F^{-1}(\beta F(E(\theta))))}.$$

The derivative

$$\begin{aligned} G'(\beta) &= F^{-1\prime}(\beta F(E(\theta)))F(E(\theta)) + \\ & \frac{F(E(\theta))}{f(F^{-1}(\beta F(E(\theta))))} + \\ & \frac{(1 - \beta F(E(\theta)))f'(F^{-1}(\beta F(E(\theta))))F^{-1'}(\beta F(E(\theta)))F(E(\theta))}{f^2(F^{-1}(\beta F(E(\theta))))} \\ &= \frac{F(E(\theta))}{f(F^{-1}(\beta F(E(\theta))))} \left\{ 2 + \frac{(1 - \beta F(E(\theta)))f'(F^{-1}(\beta F(E(\theta))))}{f^2(F^{-1}(\beta F(E(\theta))))} \right\}. \end{aligned}$$

By assumption 1,

$$2 + \frac{(1 - \beta F(E(\theta)))f'(F^{-1}(\beta F(E(\theta))))}{f^2(F^{-1}(\beta F(E(\theta))))} > 2\beta F(E(\theta)) > 0.$$

At $\beta = 0$, $\pi'_B(\widetilde{Q}_1) = E(\theta) - c - \dfrac{1}{f(0)} < 0$. Because $\pi'_B(\widetilde{Q}_1)$ is continuous and increasing in β, there exists $\beta^* \in \left(0, min\left\{\dfrac{F(c)}{F(E(\theta))}, 1\right\}\right)$ such that $\pi'_B(\widetilde{Q}_1) < 0$ for $\beta < \beta^*$.

Step 2 shows when $0 < c \leq E(\theta) - \dfrac{1}{f(0)}$, the monopolist does not ration consumers $\forall \beta \in [0, 1]$. For a given c, we first show the monopolist does not ration consumes for $\forall \beta \in [0, \dfrac{F(c)}{F(E(\theta))})$. Step 1 has shown when $\beta < \dfrac{F(c)}{F(E(\theta))}$, $1 - F(c) < q_1$. Evaluate $\pi'_B(\widetilde{Q}_1)$ at $\beta = 0$, we have $E(\theta) - c - \dfrac{1}{f(0)} > 0$. Hence, $\pi'_B(\widetilde{Q}_1) > 0$, $\forall \beta \in [0, \dfrac{F(c)}{F(E(\theta))})$. Because $\pi(q_1)$ is globally concave as is shown in the proof for Proposition 1, $\widetilde{q}_1 = \widetilde{Q}_1$, $\forall \beta \in [0, \dfrac{F(c)}{F(E(\theta))})$.

Next, we show the monopolist does not ration consumes for $\forall \beta \in \left[\dfrac{F(c)}{F(E(\theta))}, 1\right]$. When $\beta \geq \dfrac{F(c)}{F(E(\theta))}$, $\widetilde{Q}_1 \leq 1 - F(c)$. The monopolist chooses $q_1 \leq \widetilde{Q}_1$ to maximize objective function (3), with p_2 implicitly determined by equation (4). According to equation (10), evaluating $\pi'_A(q_1)$ at $q_1 = \widetilde{Q}_1$, we have $\pi'_A(\widetilde{Q}_1) = E(\theta) + (1 - \beta F(E(\theta)))\dfrac{\partial p_2}{\partial q_1}|_{q_1 = \widetilde{Q}_1}$, where $\dfrac{\partial p_2}{\partial q_1}|_{q_1 = \widetilde{Q}_1} = -\dfrac{1}{2f(p_2) + (p_2 - c)f'(p_2)}$ and p_2 is implicitly determined by $p_2 - c = \dfrac{\beta F(E(\theta)) - F(p_2)}{f(p_2)}$. When $\beta = \dfrac{F(c)}{F(E(\theta))}$, $p_2 = c$ at $q_1 = \widetilde{Q}_1 = 1 - F(c)$, and hence $\pi_A^{'}(\widetilde{Q}_1) = E(\theta) - \frac{1-F(c)}{2f(c)}$. By assumption 1, $\frac{1-F(c)}{f(c)}$ is decreasing in c. Consequently, $\pi_A^{'}(\widetilde{Q}_1) > E(\theta) - \frac{1}{f(0)} > 0$.

Take the partial derivative of $\pi_A^{'}(\widetilde{Q}_1)$ with respect to β and substitute

$$\frac{\partial p_2}{\partial q_1 \partial \beta}|_{q_1=\widetilde{Q}_1} = \frac{3f(p_2)+(p_2-c)f^{"}(p_2)}{(2f(p_2)+(p_2-c)f'(p_2))^2}\frac{\partial p_2}{\partial q_1}|_{q_1=\widetilde{Q}_1}(-F(E(\theta)))$$

, we have

$$\begin{aligned}\frac{\partial \pi_A^{'}(\widetilde{Q}_1)}{\partial \beta} &= -F(E(\theta)))\frac{\partial p_2}{\partial q_1}|_{q_1=\widetilde{Q}_1} + (-F(E(\theta)))(1-\beta F(E(\theta)))\frac{3f(p_2)+(p_2-c)f^{"}(p_2)}{(2f(p_2)+(p_2-c)f'(p_2))^2}\frac{\partial p_2}{\partial q_1}|_{q_1=\widetilde{Q}_1} \\ &= -F(E(\theta)))\frac{\partial p_2}{\partial q_1}|_{q_1=\widetilde{Q}_1}\left(1+(1-\beta F(E(\theta)))\frac{3f(p_2)+(p_2-c)f^{"}(p_2)}{(2f(p_2)+(p_2-c)f'(p_2))^2}\right).\end{aligned}$$

By assumption 1,

$$\frac{\partial \pi_A^{'}(\widetilde{Q}_1)}{\partial \beta} > -\beta F^2(E(\theta)))\frac{\partial p_2}{\partial q_1}|_{q_1=\widetilde{Q}_1} \geq 0.$$

Consequently, $\pi_A^{'}(\widetilde{Q}_1) > 0$, and $\widetilde{q}_1 = \widetilde{Q}_1$, $\forall \beta \in \left[\frac{F(c)}{F(E(\theta))}, 1\right]$. Q.E.D.

Proof for proposition 3: First consider the scenario without second-hand market. When $\overline{\theta} \leq c$, the second period price p_2 is larger than $\overline{\theta}$. No one will buy in the second period. The first period price is at most $E(\theta) + [F(p_2)E(\theta|\theta \leq p_2) + (1-F(p_2))\,p_2] = 2E(\theta)$. So, if $\max\{\overline{\theta}, 2E(\theta)\} \leq c$, the monopolist will not sell in period one either and he will make zero profit. Now, consider the scenario with secondary market. The monopolist can make a positive profit by selling to at least one consumer in period one if $\max\{\overline{\theta}, 2E(\theta)\} < c < E(\theta) + \overline{\theta}$. To see this, note that when $c \geq \max\{\overline{\theta}, 2E(\theta)\}$, $1-F(c) \leq 0$. Hence, regime A does not exist and we focus on regime B. The proof for proposition 1 has shown that the the derivative of profit function in regime B is

$$\pi_B'(q_1) = E(\theta) + F^{-1}(1-q_1) - c - \frac{q_1}{f\left(F^{-1}(1-q_1)\right)}.$$

Evaluate $\pi_B'(q_1)$ at at $q_1 = 0$, we have $E(\theta) + \overline{\theta} - c$. When $c < E(\theta) + \overline{\theta}$, $\frac{\partial \pi}{\partial q_1}|_{q_1=0} > 0$. Therefore the optimal first period output $q_1^* > 0$. Q.E.D.

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