Reputational Concerns and Price Comovements*

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Abstract

We analyze the rational expectation equilibria of a delegated portfolio management model in which two risky assets have completely independent returns and liquidity shocks. Some managers have perfect information on the assets' returns while others are uninformed and try to infer information from the prices. We show that, as long as some reasonable assumptions on the nature of the equilibrium are imposed, in a rational expectations equilibrium there is always a set of realizations of the shocks such that the returns are not revealed. In this region the prices of the two assets exhibit a strong form of comovement, as they must be identical. This occurs despite the fact that the two assets have different *ex ante* probabilities of repayment.

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1 Introduction

The growth of institutional trade in financial markets during the last decades has been widely documented. Since the management of mutual funds' portfolios is delegated to professionals, who typically have different incentives

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from investors, a large literature has studied the implications of the growth of institutional trade for the evolution of asset prices.

One phenomenon that has been osserved is the tendency to comove for the returns of assets owned by the same financial institution or group of financial institutions (see e.g. Barberis and Shleifer [2], Barberis, Shleifer and Wurgler [3], Coval and Stafford [7], and Antón and Polk [1]). This tendency to comove has been explained in two different ways. The simplest way is a mechanical effect that occurs when intermediaries which are 'large' with respect to the market try to rebalance their portfolios, when-for instance-they are hit by a liquidity shock. If a fund is forced to liquidate a significant part of its portfolio because of withdrawals and it does not want to change the composition of the portofolio then all the assets in the portfolio will experience simultanously a downward pressure on prices. Another potential explanation has to do with information transmission in a rational expectations equilibrium, as in Kodres and Pritsker [13]. They develop a model in which risk-averse investors trade assets whose returns are correlated and face liquidity shocks which may also be correlated. In their model a liquidity shock hitting one security will transmit to other securities because investor cannot separately observe liquidity and return shocks. In Kodres and Pritsker [13] when the liquidity shocks and the returns shocks are independent, there is no contagion. However, in this model we have price co-movement even when liquidity and return shocks are entirely independent.

Vayanos and Woolley [16] also consider a model in which the interaction of liquidity shocks and returns shocks generates price comovement across different assets. Their model considers a continuous time framework in which investors try to infer the ability of the managers running the funds and withdraw money from funds that perform poorly, although poor performance may be due to factors other than managerial competence. Withdrawals puts downward pressure on all the assets in the portfolio, thus producing comovement. Notice that the main goal of Vayanos and Wooley [16] is to produce a model of momentum and reversal and these phenomena appear even if their model had only one asset. Comovement is a by-product when there are multiple assets.[1].

[1]Our work is related to the literature on contagion and on the effects of managerial reputation, although most of this literature does not consider comovement. Contagion models are discussed for example in Basak and Pavlova [4], Ilyin [12] and Chakravorti and Lall [6]. The asset pricing implications of managerial reputational concerns are discussed in Cuoco and Kaniel [8] , Dasgupta and Prat [9], Dasgupta and Prat [10], and Vayanos [15]. Sharfstein and Stein [14] and Calvo and Mendoza [**?**] are examples of papers linking

In this paper we analyze a rational expectations equilibrium in which comovement appears as an equilibrium phenomenon. Our model has competitive funds, so no single fund can exert a pressure on prices, and uncorrelated return and liquidity shocks, so that in a rational expectation equilibrium *à là* Kodres and Pritsker [13] prices would be independent. Furthermore, we don't have inflows and outflows of funds: securities only last one period and then they are liquidated. A main point of departure from Kodres and Pritsker [13] is that we assume that fund managers cannot take negative positions in the assets: fund managers are given a fixed amount of money to invest and they can only buy securities with that money. Furthermore, we assume that all market participants are risk neutral.

Our starting point is the delegated portfolio management model of Guerrieri and Kondor [11]. Their model has a single risky asset and a riskless asset. The supply of the risky asset is stochastic and it is only observed *ex post*. Investors and managers are risk neutral and managerial compensation is exogenously given (it is a given fraction γ of the gross return). There are two types of fund managers, informed and uninformed; the types are private information and not observable by the investors. Informed fund managers know whether the risky assets are repaying or defaulting (they receive a perfect signal) while uninfomed fund managers don't have any information. Every investor has only one manager working for him at each period. At the end of the period, the investor observes the investment performance and updates beliefs about the manager's type. She then decides whether to fire or retain the manager.

Guerrieri and Kondor [11] analyze a rational expectations equilibrium in which prices depend on both the signal of the informed managers and the realization of the random supply (which plays the role of a liquidity shock). The information about the value of the risky asset is revealed when the liquidity shock is sufficiently high or sufficiently low, while for intermediate values of the liquidity shock the price is non-revealing.

In this environment we introduce a second risky asset. Both the return and the liquidity shock on the second asset are independent of the first. In a rational expectations equilibrium it is the case that non-revelation occurs for some realizations of the returns and of the liquidity shock. Our main result is that in the non-revelation region we have an extreme form of comovement: risky assets end up having the same price and same expected return.

The intuition for the result goes as follows. A fully revealing equilib-

contagion to herding and Wagner [17] discusses contagion due to benchmark performance schemes.

rium is impossible because of the presence of liquidity shocks which are not observed by any market participant and the impossibility of short-selling, which puts limits to arbitrage and prevents the informed managers from fully exploiting their information. Thus, in equilibrium there must be a region of realizations of returns and liquidity shocks such that the prices of the two assets do not reveal the true value. Suppose that in this region the prices of the two assets differ. When both assets repay, informed managers are going to demand exclusively the asset with the lowest price. This generates an adverse selection problem for the uninformed managers. They would get the same expected return on any of the two risky assets if such assets were actually randomly distributed, but this is not the case because whenever both assets repay the uninformed must be more likely to receive the highest priced asset. It is only when both prices are identical that this adverse selection phenomenon does not occur, as informed managers are indifferent between the two assets when they both repay.

The rest of the paper is organized as follows. In the next section we describe the model. In section 3 we define and characterize the equilibrium for the static case. We then embed the static equilibrium in a stationary equilibrium of a dynamic model in section 4. Section 5 contains concluding remarks and an appendix contains the proofs.

2 The Model

In each period each investor has a unit of funds to invest. Investors must hire fund managers in order to buy assets or have access to a risk-free technology. Both investors and managers are risk neutral and they discount the future at rate $\beta \in (0,1)$. There is a continuum of investors and managers and the measure of investors is N. Fund managers are infinitely lived and, whenever they are hired, at the beginning of each period they decide how to use the unit of capital provided by the investors. They can buy asset 0, with a safe return of R, or buy a risky asset i, with $i = 1, 2$. The return on risky asset i at time t is determined by the realization of a random variable $\widetilde{\chi}_{i,t}$ which takes values in the set $\{0,1\}$. The realization of $\widetilde{\chi}_t = (\widetilde{\chi}_{1,t}, \widetilde{\chi}_{2,t})$ is denoted $\chi_t = (\chi_{1,t}, \chi_{2,t})$. If $\chi_{i,t} = 0$ then the asset repays an amount of 1, while if $\chi_{i,t} = 1$ the asset defaults and pays zero. The random variables $\{\widetilde{\chi}_{i,t}\}_{t=0}^{\infty}$ are all independent and identically distributed, with $\Pr(\widetilde{\chi}_{i,t} = 1) = q_i$ and $q_2 > q_1$. Furthermore, each $\widetilde{\chi}_{i,t}$ is independent of all variables $\{\widetilde{\chi}_{j,\tau}\}_{\tau=0}^{\infty}$ with $j \neq i$.

Fund managers can be either informed or uninformed. Informed man-

agers observe the realization of the random vector $\widetilde{\chi}_t$ at the beginning of period t, before trading takes place. Uninformed managers receive no information.

The supply of each risky asset is modeled as in Guerrieri and Kondor [11]. The supply of asset i at time t is determined by the random variable $\widetilde{b}_{i,t}$, with realization denoted $b_{i,t}$. When the realization is $b_{i,t}$ this means that there is a mass $b_{i,t}$ of agents who wants to finance one unit of consumption and have a technology that can produce unlimited units of risky asset i. The number of units produced by each agent is enough to finance one unit of consumption, so that the aggregate amount produced is $b_{i,t}/p_{i,t}$, as long as $p_{i,t} > 0$. If $p_{i,t} = 0$ then the supply is zero. The random variables $\widetilde{b}_{i,t}$ are independently and identically distributed, have a uniform distribution with support $\left[\underline{b}, \overline{b}\right]$ and are independent from all variables $\widetilde{\chi}_{j,\tau}$ for each j and τ. We denote as $\widetilde{b}_t = \left(\widetilde{b}_{1,t}, \widetilde{b}_{2,t}\right)$ the random vector determining the supply for the assets and with $b_t = (b_{1,t}, b_{2,t})$ its realization. The vector b_t is unobservable by fund managers and investors.

Again following Guerrieri and Kondor [11], the sequence of events at each period t can be described separating what happens 'in the morning' and 'in the afternoon'.

In the 'morning' the labor market is cleared and investment decisions are made. More precisely:

- unemployed managers decide whether or not to search for a job. In order to search for a job an unemployed manager has to pay a cost κ;
- funds without a manager go to the labor market and randomly hire a manager among those who are searching.
- informed managers observe the realization of $\widetilde{\chi}_t$, while uninformed managers do not receive any information.
- both informed and uninformed managers who are employed submit vector demand schedules for the assets and the bond;
- given the realization of b_t the price vector $p_t = (p_{1,t}, p_{2,t})$ is determined to balance demand and supply in each market;
- given the prices, the assets are assigned to each fund manager according to their demand schedules.

In the 'afternoon' the realization χ_t is revealed and the investments of the managers are realized by their investors. At that point:

- managers receive a share γ of the returns;
- each investor receives an exogenous binary signal, σ_t^y about the type of the hired manager. If manager y is informed, σ_t^y is always zero, while if the manager is uninformed, $\sigma_t^y = 0$ with probability ω and $\sigma_t^y = 1$ with probability $1 - \omega$, with $\omega \in (0, 1)$;
- investors decide about firing or retaining their manager. There is also a probability $1-\delta$, with $\delta \in (0, 1)$ that any given manager is exogenously separated from the job.

A general equilibrium of the model results from the interaction of the labor market and the asset markets. Decisions made in the labor market determine the measure of informed managers present in the asset markets. In turn, this determines how much information the equilibrium price function $p_t^e(b, \chi)$ reveals and therefore how profitable it is for the investor to have an informed, rather than an uninformed, manager. In turn this determines the optimal firing rule.

The choice variables in the labor market are the firing rule ϕ_t adopted by the investor and the search decision for unemployed managers. The choice variables in the asset markets are the demand functions submitted by the managers. The market mechanism generates matching between investors and managers and equilibrium price functions in the asset markets. The interaction of these forces determines at each time t the measure N_t^I of informed managers and the value W_t^U of being employed for an uninformed manager. In the following we will focus on stationary equilibria, i.e. situations in which $\phi_t = \phi$, $p_t^e = p^e$, $N_t^I = N^I$ and $W_t^U = W^U$ for each t. The existence of such an equilibrium requires some restrictions on the parameters, that we will discuss subsequently.

We now describe more in detail how the markets work.

2.1 Asset Markets

Employed fund managers are given one unit of funds to invest and submit a demand schedule, specifying for each price vector $p = (p_1, p_2)$ which assets they are willing to buy. To keep the notation similar to the one used by Guerrieri and Kondor [11], we assume that the demand expressed by a fund manager at a given price vector is given by an element of the set

$$\Delta = \{(d_0, d_1, d_2) \,|d_i \in \{0, 1\}, \text{ for each } i = 0, 1, 2 \ \}$$

A vector $d = (d_0, d_1, d_2) \in \Delta$ is interpreted as stating the willingness (or lack of it) to buy a given asset at a certain price vector; $d_i = 1$ means that the manager is willing to buy asset i and $d_i = 0$ means that the manager is not willing to buy it (the subscript zero refers to the risk-free asset). For example, a vector $(0, 0, 1)$ indicates that the manager is willing to use the unit of funds available to buy risky asset 2 (up to an amount $1/p_2$) and nothing else. When a vector d has multiple elements equal to one then the manager is stating that she is equally happy with those assets. Thus, $d = (0, 1, 1)$ means that the manager is willing to buy either asset 1 up to an amount $1/p_1$ or asset 2 up to an amount $1/p_2$.

Let $\mathcal{D}$ be the set of functions from $\mathbb{R}^2_+$ into Δ. Then:

- an uninformed manager y chooses an element $d^y(p_1, p_2) \in \mathcal{D}$ and submits it to the auctioneer;
- an informed manager y observes the realization χ of the random vector $\widetilde{\chi}$ and submits a demand schedule $d^y(p_1, p_2 | \chi) \in \mathcal{D}$ to the auctioneer.

The aggregate demand vector will in general depend on the fraction of managers who are informed. In turn, the equilibrium price function will also depend on such fraction.

Let N^I and N^U be, respectively the mass of informed and uninformed managers employed in the period, with $N^I + N^U = N$. In equilibrium the price vector depends on the realizations of the random vectors $\widetilde{\chi}$ and $\widetilde{b}$. It can therefore be described as a price function $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0, 1\}^2 \to \left[0, \frac{1}{R}\right]^2$. Thus, in general, the realized price vector conveys information for the uninformed managers. Let $\phi(i, p^*, \chi, \sigma^y)$ be the probability that manager y is fired at the end of the period if asset i is bought when the price vector is p^*, the realization of the random vector is χ and the realization of the exogenous signal on competence is σ^y. Finally, let W^U be the value of being employed. Notice that $\delta(1 - \phi(i, p^*, \chi, \sigma^y))$ is the probability of being retained for a manager buying asset i. Define the expected utility of an uninformed manager who buys asset i when the price vector is p^* as

$$v^U(i, p^*) = E\left[\gamma \frac{1 - \widetilde{\chi}_i}{p_i} + \beta\delta\left(1 - \phi\left(i, p^*, \widetilde{\chi}, \sigma^q\right)\right) W^U \middle| p^e\left(\widetilde{b}, \widetilde{\chi}\right) = p^*\right] \tag{1}$$

and notice that the function v^U depends on the equilibrium price function $p^e(b, \chi)$, the firing rule ϕ and the future utility W^U. Define

$$D_i^U(p) = \int_{y \in N^U} d_i^y(p)\, dy$$

and

$$D_i^I(p,\chi) = \int_{y\in N^I} d_i^y(p|\,\chi)\,dy.$$

The aggregate demand for asset i at price vector p and vector χ is given by

$$D_i(p,\chi) = D_i^U(p) + D_i^I(p,\chi).$$

The excess demand vector at price vector p and vector (b,χ) is given by

$$E(p,b,\chi) = \begin{bmatrix} D_1(p,\chi) \\ D_2(p,\chi) \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$

The last definition refers to the set of players who demand a single asset. For a given (p,χ) let

$$\Xi_i(p,\chi) = \left\{ y|\, d_i^y = 1 \text{ and } d_j^y = 0 \text{ if } j \neq i \right\}$$

be the set of players who demand only asset i when the price vector is p and the realization of returns is χ, and

$$\xi_i(p,\chi) = \int_{\Xi_i(p,\chi)} d_i^y dy \qquad i = 1,2$$

as the mass of managers who demand exclusively asset i.

Let $D = (D_1, D_2) \in \mathbb{R}_+^2$ be a vector of demands and $\xi = (\xi_1, \xi_2)$ be a vector of 'exclusive' demands. A **feasible allocation rule** is a function $x(d^y, D, \xi, b) : \Delta \times \mathbb{R}_+^2 \times \mathbb{R}_+^2 \times \left[\underline{b}, \overline{b}\right]^2 \to [0,1]^3$ such that

$$\sum_{i=0}^{2} x_i(d^y, D, \xi, b)\, d_i^y = 1$$

for each $(d^y, D, \xi, b) \in \Delta \times \mathbb{R}_+^2 \times \mathbb{R}_+^2 \times \left[\underline{b}, \overline{b}\right]^2$ such that $d^y \neq (0,0,0)$. Thus, x_i can be interpreted as the probability of receiving asset i when the individual demand vector is d^y, the demand vector for the risky assets is D, the masses of managers demanding exclusively assets 1 and 2 are $\xi = (\xi_1, \xi_2)$ and the supply vector is b.

At this point we are ready to establish our equilibrium notion for the asset markets. In the following we use the convention that $\chi_0 = 0$ with probability 1 and $p_0 = \frac{1}{R}$.

Definition 1 *Take as given the collection* $\left(N^I, \phi, W^U\right)$. *A* ***rational expectations equilibrium*** *is a price function* $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2 \to \left[0, \frac{1}{R}\right]^2$, *a feasible allocation rule* $x\left(d^y, D, \xi, b\right) : \Delta \times \mathbb{R}^2_+ \times \mathbb{R}^2_+ \times \left[\underline{b}, \overline{b}\right]^2 \to [0,1]^3$ *and demand functions* $d^y\left(p|\,\chi\right)$ *for each* $y \in N^I$ *and* $d^y\left(p\right)$ *for each for each* $y \in N^U$ *such that:*

1. *For each realization* (b, χ) *the price vector* $p\left(b, \chi\right) = p^*$ *is such that all markets clear, i.e. for each asset* i

$$\int_{y \in N^I} x_i\left(\widehat{d}^y, \widehat{D}, \widehat{\xi}, b\right) \widehat{d}^y_i dy + \int_{y \in N^U} x_i\left(\widehat{d}^y, \widehat{D}, \widehat{\xi}, b\right) \widehat{d}^y_i dy - b_i \leq 0$$

$$p^*_i \left(\int_{y \in N^I_t} x_i\left(\widehat{d}^y, \widehat{D}, \widehat{\xi}, b\right) \widehat{d}^y_i dy + \int_{y \in N^U_t} x_i\left(\widehat{d}^y, \widehat{D}, \widehat{\xi}, b\right) \widehat{d}^y_i dy - b_i \right) = 0$$

   where $\widehat{d}^y = d^y\left(p^*|\,\chi\right)$ *when* $y \in N^I$, $\widehat{d}^y = d^y\left(p^*\right)$ *when* $y \in N^U$, $\widehat{D} = D\left(p^*, \chi\right)$ *and* $\widehat{\xi} = \xi\left(p^*, \chi\right)$.

2. *The demand functions of informed traders maximize their expected utility at each price vector* p^* *and* χ, *i.e. if* $d^y_i\left(p^*|\,\chi\right) = 1$ *then* $\frac{1-\chi_i}{p^*_i} \geq \frac{1-\chi_j}{p^*_j}$ *each* $j \neq i$.

3. *The demand functions of uninformed traders maximize their expected utility at each price vector* p^*, *i.e. if* $d^y_i\left(p^*\right) = 1$ *then* $v^U\left(i, p^*\right) \geq v^U\left(j, p^*\right)$ *each* $j \neq i$, *where* $v^U\left(i, p^*\right)$ *is given by expression (1) using the equilibrium price function* $p^e\left(b, \chi\right)$, *the firing rule* ϕ *and the continuation value* W^U.

As it is clear from the definition, a rational expectations equilibrium in the assets market at time t is a static concept and it is defined for a given collection $\left(N^I, \phi, W^U\right)$. In the full dynamic analysis all these objects are endogenously determined, making sure that the labor market is in equilibrium and the firing rule is optimal.

In this paper we will focus on rational expectations equilibria that satisfy some restrictions. The most important is the one described in the following definition.

Definition 2 *A rational expectations price function* $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2 \to \left[0, \frac{1}{R}\right]^2$ *is* ***compatible with excess demand schedules*** *if the following*

condition is satisfied. Let $p^* = p^e(b, \chi)$ *be the equilibrium price vector at* (b, χ)*. Suppose that at a pair* (b', χ') *we have*

$$E(p^*, b, \chi) = E\left(p^*, b', \chi'\right), \tag{2}$$

i.e. the excess demand vector at p^* *is the same at the two pairs* (b, χ) *and* (b', χ')*. Then it has to be the case that* $p^e(b, \chi) = p^e(b', \chi')$*.*

The logic of the restriction is as follows. In principle, if we know the whole aggregate demand schedule (i.e. the aggregate demand at each possible price) it is possible to infer the realization of χ, since such realization determines the shape of the demand for informed managers. In fact, we do not really need to know the whole demand schedule, as χ can be inferred by looking at 2 points. For example, consider the price vector $\widehat{p} = \left(\frac{1}{R}, \frac{1}{2R}\right)$. Let $D_2^U(\widehat{p})$ be the aggregate demand for asset 2 by uninformed managers at the price vector $\widehat{p}$. Then, if the total demand for asset 2 at the price vector $\widehat{p}$ is $D_2^U(\widehat{p}) + N^I$ it can be inferred that $\chi_2 = 0$, while if the total demand is $D_2^U(\widehat{p})$ it can be inferred that $\chi_2 = 1$. A similar price vector can be used to find out the realization of χ_1.

This seems to disclose too much information. We would like the choice of the equilibrium price vector to be based only on the observed excess demand at the equilibrium price vector, rather than on some sophisticated procedure for extracting information even at price vectors that are never observed in equilibrium. Condition (2) is a way to ensure that. It requires that when p^* is an equilibrium price vector generating a certain excess demand under a given supply vector b and a given realization χ, then it should remain an equilibrium price vector whenever the price vector generates the same excess demand at a supply vector b' and realization χ'. If we were to allow something different, that would imply that the auctioneer can select the equilibrium price vector using information other than the excess demand at that price vector[2].

To better understand the restrictions imposed by condition (2), notice that the condition is equivalent to having p^* being an equilibrium price vector whenever (b, χ) and (b', χ') are such that

$$b - b' = D^I(p^*, \chi) - D^I\left(p^*, \chi'\right). \tag{3}$$

[2]It is worth observing that at an equilibrium price vector the excess demand is not necessarily zero. Remember that the convention is that demands are expressed by signalling all the assets that the agent is willing to buy. Equilibrium requires that there is a way of allocating the assets, through the feasible allocation function x, such that the conditions in Definition 1 are satisfied.

Consider two vectors b and b' such that $b'_1 = b_1 - N^I$ and $b'_2 = b_2$ and two vectors $\chi = (0,1)$ and $\chi' = (1,1)$. At any price vector the demand of the informed managers will be zero for each risky asset whenever χ' is observed, while it will be N^I for asset 1 and zero for asset 2 whenever χ is observed. Now notice that in our example we have

$$b_1 - b'_1 = D_1^I(p^*, \chi) - D_1^I(p^*, \chi') = N^I$$

and

$$b_2 - b'_2 = D_2^I(p^*, \chi) - D_2^I(p^*, \chi') = 0.$$

Thus, condition (3) is satisfied. We should therefore have $p^e(b', \chi') = p^e(b, \chi) = p^*$, implying that observation of p^* does not fully reveal the vector χ.

In the rest of the paper we will focus on rational expectations equilibria which are compatible with excess demand schedules. Clearly, if such an equilibrium exists it cannot be fully revealing. Thus, in general, it will be valuable to employ informed rather than uninformed managers.

When the return of asset i is fully revealed by the price function then the price must be either 0 or $\frac{1}{R}$. However, non-revelation can come in many different ways. We will explore a class of equilibria similar to the one analyzed in Guerrieri and Kondor [11] for the case of one risky asset, namely equilibria in which whenever there is no full revelation of the value of any risky asset then the price function always takes the same value. We call *simple* this class of equilibria.

Definition 3 *A rational expectations price function* $p^e : \left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2 \to \left[0, \frac{1}{R}\right]^2$ *is* ***simple*** *if there is at most one pair* (p_1, p_2) *with* $p_i \in \left(0, \frac{1}{R}\right)$ $i = 1, 2$ *such that* $p^e(b, \chi) = (p_1, p_2)$.

In a simple equilibrium there is only one pair of prices which is realized in equilibrium when there is no full revelation for both assets. Notice that we still allow for the possibility that only the value of one asset is revealed while the other is not.

2.2 Labor Market

At the beginning of each period unemployed managers decide whether or not to search for a job. Search is costly: in order to be in the market a manager has to pay a cost κ. Let Z_t be the measure of managers who decide to be on the market at the beginning of time t, i.e. the supply of managers at

time t. The previous history of a manager is not observable, so there is no information on whether a given manager may be informed or uninformed. Let Z_t^I denote the mass of informed managers who are on the market at time t, and Z_t^U for the uninformed managers, with $Z_t = Z_t^I + Z_t^U$.

On the demand side we have the investors who don't have a manager, either because the previous one was fired or because there was an exogenous separation. They need to hire a new manager, since this is needed to invest their money. Let A_t be the measure of investors looking for a manager at time t. Since there is no price (managerial compensation is fixed at a fraction γ of gross return) and demand and supply are inelastic, the matching follows the Leontie f rule, that is a measure $\min\{A_t, Z_t\}$ ends up being employed. Define

$$\mu_t = \frac{\min\{A_t, Z_t\}}{Z_t}$$

as the probability that a manager searching for a job ends up being employed.

We assume that informed and uninformed managers are indistinguishable, so the probability of hiring an informed manager is $\epsilon_t = Z_t^I / \left(Z_t^I + Z_t^U\right)$. The value ϵ_t is important because it influences the firing decision of the investors. For a given probability assigned to the fact that the current manager is uninformed, whether or not it is optimal to fire the current manager depends on the probability of hiring an informed manager when going to the labor market.

After hiring has occurred, trade takes place. At the end of the period each investor observes the assets assigned to the fund manager and the realization χ_t of the vector of returns. At that point investors have to decide whether to retain or fire the manager. Typically, they will want to fire managers who are believed to be uninformed and retain managers who are believed to be informed. The new information observed by the investors at time t is the price vector p_t, the investment actually made by the manager, the return on the investment χ_t and the signal σ_t^y for manager y. The information is used to update beliefs about the managers and decide about firing or retaining them. Furthermore, there is a probability $1-\delta$ that the manager is separated from the fund for exogenous reasons (for example, the manager may relocate for family reasons). The firing rule used at time t is summarized by the function $\phi_t\left(i, p_t, \chi_t, \sigma_t^y\right)$, giving the probability of firing a manager who invested in asset i when the price vector was p_t, the realized return vector was χ_t and the exogenous signal was σ_t^y.

2.3 Assumptions

In the rest of the paper we will maintain the following assumptions that will ensure the existence of a stationary equilibrium in the asset and labor markets.

Assumption 1 *Let M^I be the measure of informed managers, M^U the measure of uninformed managers and N the measure of investors. Then $M^I < \min\left\{\underline{b}, \left(\overline{b} - \underline{b}\right)\right\}$ and $M^U > N > 2\overline{b} + M^I$.*

The assumption says that there are relatively few informed manager and in particular it is never the case that a market can clear with a demand coming only from informed managers (since $M^I < \underline{b}$). Furthermore, the mass of money to be invested is large compared to the supply of risky assets, $N > 2\overline{b} + M^I$, so that there will always be investment in the riskless bond. This will simplify the equilibrium condition, as it implies that uninformed managers have to be indifferent between the riskless bond and any risky asset with a strictly positive price. Finally, $M^U > N$ makes sure (together with Assumption 3 below) that investors are always on the short side of the managerial labor market.

Assumption 2 $q_1 \left(1 - \frac{M^I}{\Delta b}\right)^2 > \frac{\delta\omega\beta}{1+\delta\omega\beta}$

This assumption ensure that unrevealing prices are always less that $\frac{1}{R}$. It is satisfied when the probabilities of default are large enough. As in Guerrieri and Kóndor [11], risky assets may have an expected rate of return lower than the risk-free asset because uninformed managers see the risk-less asset as actually risky (it leads to firing if one of the two risky assets repays). Condition **??** makes sure that this effect is sufficiently counterbalanced by a high risk of default for the risky asset, so that the price of the risky asset remains inferior to $1/R$ in equilibrium.

Assumption 3 $\kappa < \gamma R$.

To see the role of this assumption, consider a situation in which the mass of investors trying to hire managers is greater than the mass of managers looking for a job, so that the probability of finding a job is $\mu = 1$. This situation cannot be an equilibrium because an uninformed manager could pay the search cost κ and, once hired, he would get at least γR (this can be obtained simply by investing in the riskless asset and then quitting). Since $\gamma R > \kappa$ all uninformed managers would want to enter. But then, since

$M^U > N$, the matching probability would have to be less than one. Thus, in equilibrium we must have $\mu < 1$, i.e. the mass of managers searching for a job has to be greater than the mass of funds trying to hire a manager. The presence of the exogenous probability of separation $(1-\delta)$ makes sure that in equilibrium $\mu > 0$, since firms which have exogenously lost their manager will be on the market.

Assumption 4 *Let $1-\omega$ be the probability that an uninformed manager is exogenously revealed as such. Then $1-\omega > \frac{\delta}{1+\delta}$.*

The assumption ensures that the probability assigned to the fact that a manager is informed increases in a sufficiently rapid way when the manager makes choices that are optimal *ex post* and is not revealed uninformed. In equilibrium we want this type of manager to be retained, and this happens if the probability assigned to the fact that the manager is informed is higher than the probability assigned to the fact that a manager randomly picked from the unemployment pool is informed. When $1-\omega$ is sufficiently large the probability that a manager is informed increases at a sufficiently rapid pace. For example, if $\omega = 0$, so that the signal is perfect, then the probability assigned to the fact that the manager is informed after an *ex post* optimal choice and a favorable signal would go to 1.

3 Rational Expectations Equilibria in Asset Markets

In this section we analyze the structure of rational expectations equilibria compatible with excess demand. We will take as given the quantities N^I and W^U and we will assume that the optimal firing rule ϕ is to fire any manager who does not make the *ex post* optimal choice or is revealed uninformed by the signal σ^y. We will later prove that this firing rule is optimal and we will pin down the values of N^I and W^U compatible with a stationary equilibrium.

3.1 Information Revelation in Equilibrium

An equilibrium price function $p^e(b, \chi)$ can in principle carry information about the pair (b, χ). So, the question is: how much information is revealed in a rational expectation equilibrium compatible with excess demand? Our focus will be on revelation of χ, the payoff-relevant variable.

The two extreme possibilities are that at each equilibrium price vector the actual value of χ is revealed or that nothing is revealed. We will show that neither of these cases can occur when we look at equilibria which are compatible with excess demand.

Consider the case of full revelation first. We say that a rational expectations equilibrium is **fully revealing** if the price function $p^e(b, \chi)$ is such that $p_i^e(b, \chi) = \frac{1-\chi_i}{R}$ for each i. It is worth noting that when the equilibrium is fully revealing investors are indifferent between hiring an informed or an uninformed manager, as they end up with the same allocation. In such situations managers and investors separate only for exogenous reasons and the ratio of informed and uninformed managers in the unemployment pool is irrelevant. It turns out that a fully revealing equilibrium always exists, but it is not compatible with excess demand.

Proposition 1 *For each collection* $\left(N^I, \phi, W^U\right)$ *a fully revealing rational expectations equilibrium exists. The equilibrium is not compatible with excess demand.*

The existence of a fully revealing equilibrium does not depend on the number of assets. In particular, such an equilibrium exists when $n = 1$, the case considered by Guerrieri and Kondor [11], although they focus their analysis on a partially revealing equilibrium which is simple (see Definition 3). The fully revealing equilibrium however appears quite implausible, as it requires that different equilibrium price vectors be selected at different values of χ even if the excess demand vectors are the same; thus, the fully revealing rational expectations equilibrium is *not* compatible with the excess demand schedules.

The other extreme case is the one in which no information is ever revealed. We say that a rational expectations equilibrium is **completely unrevealing** if there is a price vector p^* such that $p^e(b, \chi) = p^*$ for each pair (b, χ), i.e. the equilibrium price vector is constant. It turns out that there is no price vector p^* for which this is possible.

Proposition 2 *A completely unrevealing equilibrium does not exist.*

The intuition for the result is relatively simple and it can be better understood in the case in which there is a single risky asset. Since in equilibrium there must be demand both for the risky and the non-risky asset, some uninformed managers must demand both the risky asset and the non-risky asset. Furthermore, the price of the risky asset has to be strictly less than

$\frac{1}{R}$ and the informed managers will demand the risky asset only when it repays and the safe asset only when the risky asset defaults. The expected utility of investing in the two activities must be the same, but when an uninformed manager demands both assets an adverse selection problem arises: the probability of receiving the risky asset is lower when the asset actually repays. This adverse selection phenomenon implies that it cannot be optimal for an uninformed manager to demand both assets, thus destroying the equilibrium.

The consequence of Propositions 1 and 2 is that all equilibria compatible with excess demand must be partially revealing: in equilibrium some information is always leaked. Since the equilibrium is not fully revealing, informed managers will perform better on average than uninformed ones. On the other hand, since there is some revelation of information, uninformed managers can do better than just choosing which assets to buy at random.

3.2 General Properties of $p^e(b, \chi)$

We now investigate some general properties of any partially revealing equilibrium[3]. The next proposition states some properties that any equilibrium price function must display.

Proposition 3 *Take as given the collection* $\left(N^I, \phi, W^U\right)$*. In every rational expectations equilibrium the following must be true:*

1. *if* $p_i^e(b, \chi) = \frac{1}{R}$ *then* $\chi_i = 0$*;*
2. *if* $p_i^e(b, \chi) = 0$ *then* $\chi_i = 1$*;*
3. *if* $p_i^e(b, \chi) \in \left(0, \frac{1}{R}\right)$ *then* $v^U(i, p(b, \chi)) \geq \max_{j \neq i} v^U(j, p(b, \chi))$*;*
4. *at each vector* p *which can be obtained as a realization of* $p^e(b, \chi)$ *it must be the case that* $v^U(0, p) \geq \max_{j \neq 0} v^U(j, p)$*.*
5. *If* $p^e(b, \chi)$ *is an equilibrium price function there is no pair* (b, χ) *such that* $p_i^e(b, \chi) = \frac{1}{R}$ *for some* $i \geq 1$ *and* $p_j^e(b, \chi) \in \left(0, \frac{1}{R}\right)$ *for* $j \neq i$*.*

Notice that points (3) and (4) imply that whenever $p_1 \in \left(0, \frac{1}{R}\right)$ and $p_2 \in \left(0, \frac{1}{R}\right)$ then $v^U(0, p) = v^U(1, p) = v^U(2, p)$. In other words, the uninformed managers must be indifferent between all assets whose value is not revealed and the risk-free asset. Furthermore, point (1) implies that whenever the

[3]Notice that the equilibrium discussed in Guerrieri and Kondor [11] is partially revealing.

equilibrium price of asset i is $p_i = \frac{1}{R}$ we have $v^U(i,p) = v^U(0,p)$. Thus at any equilibrium price vector the uninformed managers must be indifferent between all the assets which have a price different from zero. The intuition for point (5) is that in order to have $p_i = \frac{1}{R}$ there must be a strictly positive demand on the part of the informed managers for asset i. But this must mean that the informed managers are unwilling to buy the other risky asset despite the fact that it has a strictly lower price. This reveals that the other asset has a return of zero and therefore the price cannot be strictly positive. Notice that it is possible to have equilibrium price vectors in which $p_i = 0$ for some asset i (so that it is revealed that $\chi_i = 1$) while the other risky asset has a price in the interval $\left(0, \frac{1}{R}\right)$, and is thus non-revealing. However, if $p_i = \frac{1}{R}$ for some i then full revelation must occur: all prices are either 0 or $\frac{1}{R}$. We can now establish the first important result.

Proposition 4 *There is no simple equilibrium in which, for some (b,χ), it holds that $\frac{1}{R} > p_1^e(b,\chi) > p_2^e(b,\chi) > 0$ or $\frac{1}{R} > p_2^e(b,\chi) > p_1^e(b,\chi) > 0$.*

The intuition is very similar to the one behind Proposition 2. When $p_1 > p_2$ the uninformed managers tend to receive asset 1 with higher probability when $\chi = (0,0)$ rather than when $\chi = (0,1)$. This adverse selection problem implies that the expected value of asset 1 conditional on receiving asset 1 is lower than the expected value of receiving asset 0. Thus, uninformed managers are better off not demanding asset 1 at the pair of prices (p_1, p_2). But this makes it impossible for the market for asset 1 to clear.

3.3 Building a Partially Revealing Equilibrium

Proposition (4) implies that a simple equilibrium can exist only if the two risky assets have the same price whenever there is no full revelation. In other words, there will be a set of values of (b,χ) for which the price vector is (p,p), with $p \in \left(0, \frac{1}{R}\right)$, while outside the set there will be revelation of at least one asset. In order to further explore the nature of equilibrium we start from a somewhat obvious observation that we state without proof.

Lemma 1 *Suppose that there is an equilibrium in which the price function $p^e(b,\chi)$ is such that $p_1^e(b,\chi) = p_2^e(b,\chi) = p$ and $p \in \left(0, \frac{1}{R}\right)$ for some subset of $\left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2$. Then it must be the case that*

$$\Pr(\chi_1 = 0 | (p,p)) = \Pr(\chi_2 = 0 | (p,p)).$$

In equilibrium the two assets must generate the same expected utility for the uninformed managers when the price vector is (p,p). Since the two assets have the same price, so they are both the lowest priced risky asset, the expected utility of each asset is $(1-f_i)\left(\frac{\gamma}{p}+\beta\omega\delta W^U\right)$, where

$$f_i = \Pr\left(\chi_i = 0 | \left(p,p\right)\right).$$

To generate the same utility we must have $f_1 = f_2$. If we call f the common probability we must also have

$$(1-f)\left(\frac{\gamma}{p}+\beta\omega\delta W^U\right) = \gamma R + \beta\omega\delta f^2 W^U.$$

Define now

$$r = \frac{N^I}{\overline{b}-\underline{b}}$$

and observe that, because of Assumption 1, we have $r \in (0,1)$. The ratio r measures the impact of informed traders and we will see that it plays an important role in the construction of the equilibrium.

The key observation is that when the price pair (p,p) occurs in equilibrium then the informed traders are indifferent between the two assets when $\chi = (0,0)$. If they were to choose both assets as part of their demand then the posterior probabilities could not be equal. But by Lemma 1 they must be equal. The equilibrium therefore requires that when $\chi = (0,0)$ there is some asymmetry in the demand for the two assets, so that posterior beliefs end up being the same. Let α be the fraction of informed traders demanding asset 1 when the prices are (p,p) and $\chi = (0,0)$, and β the similar fraction for asset 2. The values α and β are chosen so that $\Pr\left(\chi_1 = 0 | \left(p,p\right)\right) = \Pr\left(\chi_2 = 0 | \left(p,p\right)\right)$. The existence of the values α and β that makes this possible is not obvious and requires conditions on the parameters. This leads to the following proposition.

Proposition 5 *A simple equilibrium in which* $p_1^e\left(b,\chi\right) = p_2^e\left(b,\chi\right) = p$ *and* $p \in \left(0, \frac{1}{R}\right)$ *for some subset of* $\left[\underline{b},\overline{b}\right]^2 \times \{0,1\}^2$ *is possible only if*

$$rq_2\left(1-q_1\right) \geq q_2 - q_1. \tag{4}$$

Condition (4) is satisfied when r is sufficiently high or when the difference $q_2 - q_1$ is sufficiently small. When $r = 1$ the condition is always satisfied, and so it is when $q_2 = q_1$. The economic intuition is as follows. We want to move from a situation in which the probabilities of repayment are different

($q_2 > q_1$) to a situation in which the probabilities are equal ($f_1 = f_2$). In order to do that we need a sufficient mass of informed managers. When the priors are very close then a slight asymmetry in the demands for the assets by informed managers is sufficient to achieve equality. When the initial difference is high we need a more robust presence of informed managers, which is equivalent to a higher r.

We are now in a position to characterize a simple equilibrium. In equilibrium there are values of (b, χ) for which no revelation occurs and in that case the two assets will have the same price. There will also be areas in which full revelation occurs, with both assets having prices reflecting their fundamentals, and areas in which one asset is revealed not paying while the other is not fully revealed.

Proposition 6 *Suppose that inequality (4) is satisfied. Then there is a continuum of simple equilibria compatible with excess demand in which the equilibrium price function takes the following values:*

- *no revelation and perfect comovement: the prices are given by* (p, p)*, where* $p = \frac{1-f}{R+(f^2-(1-f))\beta\omega\delta\frac{W^U}{\gamma}}$*;*
- *partial revelation: the prices are given either by* $(0, p_2)$ *or* $(p_1, 0)$*, where* $p_i = \frac{1-q_i}{R+(2q_i-1)\beta\omega\delta\frac{W^U}{\gamma}}$*;*
- *full revelation.*

In equilibrium, when $\chi = (0, 0)$ *a fraction* α *of informed traders demand asset 1 and a fraction* β *demand asset 2, with* $\alpha \in [0, 1]$*,* $\beta \in [0, 1]$ *and* $\alpha + \beta \in [1, 2]$*. The value* f *is the conditional probability of failure for an asset when the realized price pair is* (p, p) *and it depends on* α *and* β*.*

In the appendix we explain in detail the shapes of the different regions. The basic idea is to generalize the equilibrium structure in Guerrieri and Kondor [11]. In their setting with a single risky asset, non-revelation occurs when either the asset repays and the supply of bond is sufficiently high ($\chi = 0$ and $b \in \left[\underline{b} + N^I, \overline{b}\right]$) or the asset does not repay and the supply of the bond is sufficiently low ($\chi = 1$ and $b \in \left[\underline{b}, \overline{b} - N^I\right]$).

With two assets we have a similar structure but we have to make sure that the regions of non-revelation are determined in such a way that the posterior probabilities are equal. So, for example, when both assets repay ($\chi = (0, 0)$) the non-revelation region is given by $\left[\underline{b} + \alpha N^I, \overline{b}\right] \times \left[\underline{b} + \beta N^I, \overline{b}\right]$, while

when both assets fail the non-revelation region is $\left[\underline{b}, \overline{b} - \alpha N^I\right] \times \left[\underline{b}, \overline{b} - \beta N^I\right]$. The choice of α and β, i.e. the weight put in the demand of asset 1 and asset 2 by informed managers when $\chi = (0, 0)$, determines the boundaries of the non-revelation region and therefore the conditional probability of default for each asset when (p, p) is observed. Condition (4) makes sure that it is possible to choose α and β so that the probability of default is the same for the two assets.

It is worth noticing that when there is partial revelation, i.e. one asset is revealed as defaulting while uncertainty remains on the other asset, the price of the asset is exactly the same as in Guerrieri and Kondor [11].

4 Stationary Equilibrium

Up to now the analysis has taken the mass of informed managers N^I, the firing rule and the present value W^U of the utility of being employed for uninformed managers as given. The values N^I and W^U can be endogenized and the firing rule made optimal in a stationary equilibrium, by slightly adjusting the analysis in Guerrieri and Kondor [11]. This section shows how this is done, so that the paper is self contained.

4.1 Labor Market and Determination of N^I and W^U

Let W^U be the present value of being employed for an uninformed manager. If the uninformed manager stays out of the labor market, current period utility is zero. If instead the manager decides to search for a job then, as explained in subsection 2.2, the probability of finding one is $\mu_t = \min\{A_t, Z_t\}/Z_t$. Call $\widehat{W}_t^U$ the present value of an *unemployed* uninformed manager making the optimal search decision at time t. The the following relation must hold:

$$\widehat{W}_t^U = \max\left\{\beta \widehat{W}_{t+1}^U, \mu_t W_t^U + (1 - \mu_t)\,\beta \widehat{W}_{t+1}^U - \kappa\right\} \tag{5}$$

where $\beta \widehat{W}_{t+1}^U$ is the utility obtained if no search is made in the current period (thus not paying the cost κ and obtaining zero in the current period).

In a stationary equilibrium $W_t^U = W^U$ and $\widehat{W}_t^U = \widehat{W}^U$ for each t. Furthermore, in a stationary equilibrium in which the probability of finding employment is μ whenever the search cost κ is paid we must have

$$\mu W^U + (1 - \mu)\,\beta \widehat{W}^U - \kappa = 0. \tag{6}$$

The reason is that with $\mu W^U + (1-\mu)\,\beta\widehat{W}^U - \kappa > 0$ we would have all uninformed managers entering, which is impossible as there is a large mass of uninformed managers and the probability of obtaining employment would be too low. On the other hand if $\mu W^U + (1-\mu)\,\beta\widehat{W}^U - \kappa < 0$ then only informed managers can possibly be in the market and the firing rule cannot be optimal (any manager who has a probability of being informed less than 1 should be fired).

Equation (5) and (6) imply that in a stationary equilibrium $\widehat{W}^U = 0$ and therefore

$$\mu W^U = \kappa. \tag{7}$$

Notice further that the utility of being employed for an informed manager must be higher than for an uninformed manager, since the probability of losing the job is lower. Thus $W^I > W^U$. Since the probability of getting hired is the same for informed and uninformed managers, equation (7) implies $\mu W^I - \kappa > 0$, so that it is always optimal for an unemployed informed manager to search for a job.

Let λ be the probability that an uninformed manager makes a choice that does not lead to being fired. This will happen either because the equilibrium is revealing or because the equilibrium is unrevealing but the uninformed manager makes by chance the correct choice. Furthermore, in both cases, the uninformed manager needs the message σ^y not to reveal that she is uninformed. The probability λ is an increasing function of N^I, the number of informed managers, since the larger is N^I the higher is the probability that the equilibrium price vector will be fully revealing and therefore the uninformed managers will be able to make the correct choice. Furthermore, in a stationary equilibrium the number N^I must satisfy

$$(1-\delta)\,N^I = \mu\left(M^I - \delta N^I\right).$$

The reason is that, as previously observed, in an equilibrium characterized by the free entry condition (7) for the uninformed manager, all informed managers must have a strictly positive utility from searching. In any given moment, the number of unemployed informed managers is M^I minus the ones who were employed in the previous period and retained their job, i.e. δN^I. Thus $M^I - \delta N^I$ is the mass of informed unemployed managers who search and $\mu\left(M^I - \delta N^I\right)$ is the number of informed managers hired in any given period. In order to keep the mass of informed managers constant, this number must equal the number of informed managers departing in each period. Since an informed manager departs only for exogenous reasons, the

number is $(1-\delta)N^I$. Thus, when the matching probability is μ we have

$$N^I = \frac{\mu M^I}{1-\delta+\delta\mu} \tag{8}$$

Thus, N^I is an increasing function of μ. This in turn implies that λ is also an increasing function of μ.

Let $\lambda(\mu)$ denote the function that describes the probability of being retained for an uninformed manager when buying the riskless asset. Notice that $\lambda(\mu)$ is increasing in μ and $\lambda(\mu) \in [0,1]$ whenever $\mu \in [0,1]$

Since choosing the riskless asset must always be an optimal choice for the uninformed manager we must have

$$W^U = \gamma R + \beta\delta\omega\lambda(\mu) W^U \qquad \rightarrow \qquad W^U = \frac{\gamma R}{1-\beta\delta\omega\lambda(\mu)}. \tag{9}$$

Using the free-entry condition (7) and the expression for W^U in (9) we obtain the following equation to be solved for μ:

$$\mu\gamma R = \kappa\left(1-\beta\delta\omega\lambda(\mu)\right). \tag{10}$$

The LHS is continuous and strictly increasing in μ while the RHS is continuous and decreasing in μ. Furthermore, at $\mu = 0$ the LHS is strictly lower than the RHS and at $\mu = 1$ the LHS is strictly higher than the RHS (since we assumed $\gamma R \geq \kappa$). Thus, a unique value $\mu^* \in (0,1)$ exists. In turn, this determines a unique value N^I from equation (8) and a unique value for W^U from equation (9).

4.2 Firing Rule Optimality

The last step is to show that the firing rule (i.e. fire a manager only when she is revealed uninformed, retain the manager otherwise) is optimal. It is obvious that, as long as there is a strictly positive percentage of informed managers looking for a job, it is optimal to fire a manager who is considered uninformed with probability 1. What we need to prove is that it is never the case that it is optimal to fire a manager who has not been revealed uninformed. This is the case if the probability that a manager is informed is higher than the fraction of informed in the unemployment pool, as stated by the following prposition.

Proposition 7 *The firing rule is optimal.*

In a stationary equilibrium there is always a strictly positive fraction of informed managers in the unemployment pool, due to the exogenous rate of separation δ and to the fact that the free-entry condition for uninformed implies that it is stricty optimal for informed managers to search for a job. This immediately implies that it is optimal to fire a manager who has been proved uninformed. To complete the prove of optimality we also need to show that it is never optimal to fire a manager who has *not* been proved uninformed. This is true if the probability assigned to the fact that a manager (who has made no mistakes) is informed is greater than the fraction of informed managers present in the unemployment pool. Essentially, what is required is that the probability that a manager is informed goeas up sufficiently fast when she does not make a mistake. The proof shows that when Assumption 4 is satisfied this is the case.

5 Conclusion

This paper explores a rational expectation model in which two assets with independent returns and liquidity shocks are present. The equilibrium exhibits a strong form of price comovement: unless there is full revelation of the information on the assets' returns, the prices of the two assets are the same. This happens despite the fact that *ex ante* the two assets have different distributions.

The intuition for the result is that in an equilibrium in which the assets have non-revealing but different prices, informed managers will buy exclusively the asset with the lower price when both assets are repaying. Thus, the uninformed managers face an adverse selection problem that prevents the existence of such equilibrium. It is only when the informed managers are indifferent between the two assets when they both repay, something that happens only if they have the same price, that an equilibrium becomes possible.

This somewhat extreme form of comevement occurs because all the fund managers are evaluated looking at the performance of all assets. An extension of the model may consider the case in which there are both specialized and general funds. Specialized funds are restricted to buy certain classes of assets and the managers are evaluated only looking at the *ex post* performance of those assets. In such a model the managers of the specialized funds do not face the same type of adverse selection problem that managers of the general funds face, so equilibria with different unrevealing prices become possible.

Appendix

Proof of Proposition 1. In a fully revealing rational expectations equilibrium the price function $p^e(b,\chi)$ is given by $p_i^e(b,\chi) = \frac{1-\chi_i}{R}$, so that in equilibrium any asset has a price which is either 0 or $\frac{1}{R}$. Prices different from 0 or $\frac{1}{R}$ are not observed in equilibrium, so that the probability distribution held by uninformed agents at a price $p_i \notin \left\{0, \frac{1}{R}\right\}$ is undetermined. We specify that whenever $p_i \notin \left\{0, \frac{1}{R}\right\}$ uninformed managers believe $\chi_i = 1$ with probability 1, i.e. they are certain that the asset will default.

Given this price function and beliefs, a demand function that maximizes the expected utility of an uninformed manager is given by

$$d_i^U(p) = \begin{cases} 1 & \text{if } p_i = \frac{1}{R} \\ 0 & \text{otherwise} \end{cases}$$

for each asset $i = 0, 1, 2$. The demand function of the informed can be described as follows. For any given realization χ, let

$$P(\chi) = \{i \,|\chi_i = 0\}$$

be the set of assets which are going to repay. Then

$$d_i^I(p) = \begin{cases} 1 & \text{if } \chi_i = 0 \text{ and } p_i \leq p_j \text{ for each } j \in P(\chi) \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

i.e. the informed manager demands asset i if and only if the asset is not in default and it has the lowest price among the assets which are not in default (we maintain the convention that $\chi_0 = 0$ with probability 1).

It is clear that, given the price function and the specified beliefs for out-of-equilibrium prices, the demand functions maximize the expected utility of both informed and uninformed managers. With these demand functions:

- since $N^I \leq M^I < \underline{b}$, no market can be in equilibrium unless there is demand on the part of the uninformed managers;
- the price cannot be 0 for a repaying asset, since in that case the demand on the part of the informed would be strictly positive and the supply would be zero;
- thus, the only possible equilibrium price vector is such that $p_i = \frac{1-\chi_i}{R}$ for each i; no other equilibria are possible.

To show that the price function $p_i = \frac{1-\chi_i}{R}$ is in fact an equilibrium we have to specify the allocation rule x. For a given vector $d^y \in \Delta$ define $\delta^1(d^y) = \{i \,|\, d_i^y = 1\}$ and let $\{i_1, i_2, \ldots, i_m\}$ be an enumeration of $\delta^1(d^y)$. We set $x_i = 0$ if $i \notin \delta^1(d)$. For assets in the set $\delta^1(d^y)$ we define x_i as follows:

- If $\delta^1(d^y)$ has a single element i_1 then $x_{i_1} = 1$.
- If $\delta^1(d^y)$ has multiple elements we define recursively x_{i_k} as follows:
  - $x_{i_m} = \frac{b_{i_m}}{N}$;
  - $x_{i_k} = \min\left\{\frac{b_{i_k}}{N}, 1 - \sum_{j=k+1}^{m} x_{i_j}\right\}$;
  - $x_{i_1} = 1 - \sum_{j=2}^{m} x_{i_j}$.

It can be readily checked that the allocation rule satisfies $\sum_{i=0}^{2} x_i d_i^y = 1$ for each d^y and b.

We can now check that this allocation rule clears the markets at each possible equilibrium price vector. Remember that in equilibrium we have $d_i^U = 1$ only when $p_i = \frac{1}{R}$, and $p_i = \frac{1}{R}$ if and only if $\chi_i = 0$. All managers, informed and uninformed have the same demand at an equilibrium point. Thus the aggregate demand is N (the entire mass of traders) if $p_i = \frac{1}{R}$ and 0 if $p_i = 0$. Furthermore, the set $\delta^1(d^y)$ has always as first element the safe asset, i.e. $i_1 = 0$ and includes the assets with price $\frac{1}{R}$. Since, by assumption $N > 2\bar{b}$ we have that at an equilibrium point $x_i = \frac{b_i}{N} \in (0,1)$ when $p_i = \frac{1}{R}$ and $x_i = 0$ when $p_i = 0$, thus clearing all markets.

That this equilibrium is not compatible with excess demand can be seen immediately considering two vectors (b, χ) and (b', χ') such that $b'_1 = b_1 - N^I$, $\chi_1 = 0$ and $\chi'_1 = 1$ (see the discussion after Definition 2). ■

Proof of Proposition 2. Suppose that the equilibrium price function is such that $p^e(b, \chi) = p^*$ for each vector (b, χ) for some vector p^*. In an equilibrium in which prices are constant and do not depend on (b, χ) the probability conditional on prices must be equal to the prior probabilities, i.e. $\Pr(\chi_i = 1 \,|\, p^*) = q_i$ for each i. In order to have positive demand for each asset, the expected utility of investing in each asset must be the same. We have

$$\begin{aligned} v^U(i, p^*) &= E\left[\gamma \frac{1 - \widetilde{\chi}_i}{p_i^*} + \beta\delta\left(1 - \phi\left(i, \widetilde{\chi}, p^*, \sigma^y\right)\right) W^U \,\middle|\, p^e\left(\widetilde{b}, \widetilde{\chi}\right) = p^*\right] \\ &= \gamma \frac{1 - q_i}{p_i} + \beta\delta\omega \Pr(i \text{ ex post optimal}) W^U \end{aligned}$$

where we have assumed that the firing rule is that a manager is retained only if the exogenous signal σ^y does not reveal that the manager is uninformed (probability ω) and the choice of i turns out to be optimal *ex post*. For $i = 0$ the probability that the choice is *ex post* optimal is $q_1 q_2$, i.e. the probability that all risky assets will fail. Thus, we have

$$v^U(0, p^*) = \gamma R + \beta\delta\omega q_1 q_2 W^U$$

Consider now the possible pricing of the risky assets. We start observing that we can rule out the case $p_1^* = p_2^*$. In this case $\Pr(i \text{ ex post optimal}) = 1 - q_i$, so the prices must be

$$p_1^* = \frac{1 - q_1}{R + (q_1 q_2 - (1 - q_1)) \frac{\beta\delta\omega}{\gamma} W^U}$$

$$p_2^* = \frac{1 - q_2}{R + (q_1 q_2 - (1 - q_2)) \frac{\beta\delta\omega}{\gamma} W^U},$$

but this implies $p_1^* \neq p_2^*$, a contradiction.

Thus suppose $p_1^* > p_2^*$ (the case $p_2^* > p_1^*$ is symmetric). In this case

$$v^U(1, p^*) = \gamma \frac{1 - q_1}{p_1} + \beta\delta\omega(1 - q_1) q_2 W^U$$

$$v^U(2, p^*) = \gamma \frac{1 - q_2}{p_2} + \beta\delta\omega(1 - q_2) W^U$$

and the prices must be

$$p_1^* = \frac{1 - q_1}{R + (q_1 q_2 - (1 - q_1) q_2) \frac{\beta\delta\omega}{\gamma} W^U} \tag{12}$$

$$p_2^* = \frac{1 - q_2}{R + (q_1 q_2 - (1 - q_2)) \frac{\beta\delta\omega}{\gamma} W^U} \tag{13}$$

If the prices given by (12) and (13) are such that $p_1^* \leq p_2^*$ then there is no equilibrium of this sort. Thus, suppose that in fact the parameters are such that $p_1^* > p_2^*$. In equilibrium the demand functions of the informed managers are given by (11). Since there are no ties among prices, informed managers always demand at most one risky asset. For a given vector (b, χ) the *ex post* utility of manager y with demand d^y is

$$u(d^y, (b, \chi)) = x_0(d^y, b, \chi) \left(\gamma R + \beta\delta\omega\chi_1\chi_2 W^U\right)$$

$$+x_1\left(d^y,b,\chi\right)\left(1-\chi_1\right)\left(\frac{\gamma}{p_1^*}+\beta\delta\omega\chi_2W^U\right)+x_2\left(d^y,b,\chi\right)\left(1-\chi_2\right)\left(\frac{\gamma}{p_2^*}+\beta\delta\omega W^U\right),$$

where $x_i\left(d^y,b,\chi\right)$ is the probability of receiving asset i when (b,χ) occurs and the demand is d^y.

Now notice that whenever asset i is the lowest priced repaying asset, a quantity N^I must be allocated to the informed managers, as this is the only asset that they demand. Integrating over N^U the quantity $x_1\left(d^y,b,\chi\right)$ we therefore have

$$\int_{y\in N^U}x_1\left(d^y,b,\chi\right)dy=b_1-N^I\left(1-\chi_1\right)\chi_2 \tag{14}$$

and integrating over N^U the quantity $x_2\left(d^y,b,\chi\right)$ we have

$$\int_{y\in N^U}x_2\left(d^y,b,\chi\right)dy=b_2-N^I\left(1-\chi_2\right). \tag{15}$$

Finally, the quantity of riskless bond allocated to uninformed managers is determined residually as

$$\int_{y\in N^U}x_0\left(d^y,b,\chi\right)dy=N^U-\left(b_1-N^I\left(1-\chi_1\right)\chi_2\right)-\left(b_2-N^I\left(1-\chi_2\right)\right) \tag{16}$$

Using (14), (15) and (16), by integrating $u\left(d^y,b,\chi\right)$ over N^U we obtain

$$\int_{y\in N^U}u\left(d^y,b,\chi\right)dy=$$

$$\begin{aligned}&\left(N^U-\left(b_1-N^I\left(1-\chi_1\right)\chi_2\right)-\left(b_2-N^I\left(1-\chi_2\right)\right)\right)\left(\gamma R+\beta\delta\omega\chi_1\chi_2W^U\right)\\&+\left(b_1-N^I\left(1-\chi_1\right)\chi_2\right)\left(1-\chi_1\right)\left(\frac{\gamma}{p_1^*}+\beta\delta\omega\chi_2W^U\right)\\&+\left(b_2-N^I\left(1-\chi_2\right)\right)\left(1-\chi_2\right)\left(\frac{\gamma}{p_2^*}+\beta\delta\omega W^U\right)\end{aligned}$$

Taking expectation with respect to χ, and using the fact that b and χ are independent and $v^*=v^U\left(i,p^*\right)$ for each i, we have

$$\int_{y\in N^U}E_\chi\left[u\left(d^y,b,\chi\right)\right]dy=N^Uv^*$$

$$-N^IE\chi\left[\left(1-\chi_1\right)\chi_2\left(\left(1-\chi_1\right)\left(\frac{\gamma}{p_1^*}+\beta\delta\omega\chi_2W^U\right)-\left(\gamma R+\beta\delta\omega\chi_1\chi_2W^U\right)\right)\right]$$

$$-N^I E\chi\left[(1-\chi_2)\left((1-\chi_2)\left(\frac{\gamma}{p_2^*}+\beta\delta\omega W^U\right)-\left(\gamma R+\beta\delta\omega\chi_1\chi_2 W^U\right)\right)\right]$$

It can be readily checked that the second and third term on the right hand side are negative. For example, the second term is non-zero only when $\chi_1 = 0$ and $\chi_2 = 1$. In that case the term in the square parenthesis is $\frac{\gamma}{p_1^*} + \beta\delta\omega W^U - \gamma R$ which is strictly positive since $1/R > p_1^*$, and multiplication by $-N^I$ yields a negative value.

This means that there must be a positive mass of uninformed managers who obtain an expected utility lower than v^*. This cannot be the case in equilibrium. ■

Proof of Proposition 3. Remember that the supply of risk-free bonds is infinitely elastic at the price $\frac{1}{R}$ and that in equilibrium $N^I \leq M^I < \underline{b}$, so that equilibrium at any non-zero price for a risky asset is possible only if there is demand from the uninformed managers.

1. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ with $p_i^e(b, \chi) = \frac{1}{R}$. If $\Pr(\chi_i = 0 | p^e(b, \chi)) < 1$ then the investment is strictly dominated by the investment in the riskless asset for the uninformed managers, so their demand for asset i at that price vector is zero. But then $p_i^e = \frac{1}{R}$ cannot be part of an equilibrium price vector since the demand for asset i is at most N^I and it is therefore strictly less than supply.

2. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ with $p_i^e(b, \chi) = 0$. If $\Pr(\chi_i = 1 | p^e(b, \chi)) < 1$ then the demand on asset i by the uninformed manager would be infinity, thus violating the equilibrium condition.

3. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ with $p_i^e(b, \chi) \in \left(0, \frac{1}{R}\right)$. Then demand must be equal to supply for asset i and this is possible only if there is a strictly positive demand by uninformed managers. In turn, this is possible only if $v^U(i, p) \geq v^U(j, p)$ for each $j \neq i$.

4. Suppose that at a vector (b, χ) the equilibrium price vector is $p^e(b, \chi)$ and $\max_{j \neq 0} v^U(j, p) > v^U(0, p)$. Then demand for the risk-free asset can only come from the informed managers and the uninformed managers will only demand risky assets. But since $N - M^I > 2\bar{b}$, it is impossible to reach equilibrium in all markets for risky assets.

5. Suppose that at a vector (b, χ) the equilibrium price vector is $p^* = p^e(b, \chi)$ with $p_i^* = \frac{1}{R}$ for some $i \geq 1$ and $p_j^* \in \left(0, \frac{1}{R}\right)$ for $j \notin \{0, i\}$. It must be the case that the informed managers are demanding asset i, thus revealing that $\chi_i = 0$. If $p_j^* \in \left(0, \frac{1}{R}\right)$ this means that informed managers must demand asset j with strictly positive probability, i.e. that $\chi_j = 0$ with strictly positive probability when the price vector is p^*. But this is impossible, since in this case the informed managers would not demand asset i (which has a higher price), thus making it impossible to have $p_i^* = \frac{1}{R}$. So p_j^* must be either 0 or $\frac{1}{R}$. ■

Proof of Proposition 4. We break down the proof in two steps.

Step 1. If $b_i \geq \underline{b} + N^I$ then $\chi_i = 0$ cannot be revealed. This is proved by contradiction. Consider wlog asset 1 and suppose there is a pair $((b_1, b_2), (\chi_1, \chi_2))$ such that $b_1 \geq \underline{b} + N^I$, $\chi_1 = 0$, and $p_1 = \frac{1}{R}$. First notice that by point (5) of Proposition (3), full revelation of χ_2 must also occur. Suppose first $(\chi_1 = 0, \chi_2 = 1)$ and consider the pair $(b', \chi') = \left(\left(b_1 - N^I, b_2\right), (1, 1)\right)$. By compatibility with excess demand the price must be the same at (b', χ') and at (b, χ). But this is impossible, since we would have $p_2 = \frac{1}{R}$ and $\chi_2 = 1$. Next suppose $(\chi_1, \chi_2) = (0, 0)$. Let (α, β) be the demand for the risky assets on the part of the informed managers when prices are $\left(\frac{1}{R}, \frac{1}{R}\right)$ and $(\chi_1, \chi_2) = (0, 0)$, with $\alpha \in \left[0, N^I\right]$ and $\beta \in \left[0, N^I\right]$. We distinguish two cases.

a) β is such that $b_2 \geq \underline{b} + \beta$. In that case the excess demand would be exactly the same at $(b', \chi') = ((b_1 - \alpha, b_2 - \beta), (1, 1))$.

b) β is such that $b_2 < \underline{b} + \beta$. In that case the excess demand would be exactly the same at $(b', \chi') = \left(\left(b_1 - \alpha, b_2 + N^I - \beta\right), (1, 0)\right)$. Notice that $b_2' = b_2 + N^I - \beta$ is in the interval $\left[\underline{b}, \underline{b} + N^I\right]$ and it is therefore feasible.

We conclude that whenever $b_i \geq \underline{b} + N^I$ and $\chi_i = 0$ the price function must be unrevealing, i.e. there must be vector (b', χ') with $\chi_i' = 1$ such that $p^e(b, \chi) = p^e(b', \chi')$.

Step 2. Let

$$u_1(\chi | (p_1, p_2)) = (1 - \chi_1)\left(\frac{\gamma}{p_1} + \chi_2 \beta \delta \omega W^U\right)$$

be the *ex post* utility of an uninformed manager who receives asset 1 when the prices are (p_1, p_2) with $p_1 > p_2$ (the case $p_1 < p_2$ is similar) and (χ_1, χ_2) realizes; observe that $u_1\left((1, \chi_2)\middle|\,(p_1, p_2)\right) = 0$. Let

$$f_{ij} = \Pr\left(\widetilde{\chi}_1 = i, \widetilde{\chi}_2 = j\middle|\,(p_1, p_2)\right)$$

be the conditional probability of $\widetilde{\chi} = (i, j)$ when the prices are (p_1, p_2). In equilibrium it has to be

$$f_{00}u_1\left((0,0)\middle|\,(p_1, p_2)\right) + f_{01}u_1\left((0,1)\middle|\,(p_1, p_2)\right) = v^*$$

where v^* is the expected utility obtained demanding the safe asset only.

Let $x_1\left(b, \chi\middle|\,(p_1, p_2)\right)$ be the quantity of asset 1 given to uninformed managers when the supply of the risky assets is $b = (b_1, b_2)$, the realization is χ and prices are (p_1, p_2). Since when $\chi = (0, 0)$ the informed managers demand asset 2 only, in equilibrium it has to be the case that $x_1\left(b, (0,0)\middle|\,(p_1, p_2)\right) = b_1$. On the other hand, when $\chi = (0, 1)$ the informed managers demand asset 1 only, so $x_1\left(b, (0,1)\middle|\,(p_1, p_2)\right) = b_1 - N^I$. Define

$$\overline{x}_1^{00} = E\left[x_1\left(b, (0,0)\middle|\,(p_1, p_2)\right)\right]$$

and

$$\overline{x}_1^{01} = E\left[x_1\left(b, (0,1)\middle|\,(p_1, p_2)\right)\right]$$

We want to show that

$$\overline{x}_1^{00} > \overline{x}_1^{01}.$$

We first observe that

$$\overline{x}_1^{00} = E\left[x_1\left(b, (0,0)\middle|\,(p_1, p_2)\right)\right] = E\left[b_1\middle|\,\chi = (0,0), (p_1, p_2)\right] \geq \frac{\underline{b} + \overline{b}}{2}$$

since, by step 1, the no-revelation region must include the upper interval $\left[\underline{b} + N^I, \overline{b}\right]$. Furthermore

$$\overline{x}_1^{01} = E\left[x_1\left(b, (0,1)\middle|\,(p_1, p_2)\right)\right] = E\left[b_1 - N^I\middle|\,\chi = (0,1), (p_1, p_2)\right] \leq \frac{\underline{b} + \overline{b} - N^I}{2}$$

since $E\left[b_1\middle|\,\chi = (0,1), (p_1, p_2)\right] \leq \frac{\underline{b} + \overline{b} + N^I}{2}$, where the highest value is attained when the no-revelation region is exactly $\left[\underline{b} + N^I, \overline{b}\right]$. We conclude

$$\overline{x}_1^{00} > \overline{x}_1^{01}$$

Let

$$\overline{x}_1 = f_{00}\overline{x}_1^{00} + f_{01}\overline{x}_1^{01}$$

be the expected quantity of asset 1 received by uninformed managers who are willing to buy asset 1 when prices are (p_1, p_2) and $\chi_1 = 0$. The expected utility *conditional on receiving asset* 1 by an uninformed manager is

$$\frac{f_{00}\overline{x}_1^{00}}{\overline{x}_1} u_1\left(\left(0,0\right)|\left(p_1,p_2\right)\right) + \frac{f_{01}\overline{x}_1^{01}}{\overline{x}_1} u_1\left(\left(0,1\right)|\left(p_1,p_2\right)\right) <$$

$$f_{00}u_1\left(\left(0,0\right)|\left(p_1,p_2\right)\right) + f_{01}u_1\left(\left(0,1\right)|\left(p_1,p_2\right)\right) = v^*.$$

But this means that for at least some uninformed agents demanding asset 1 at prices (p_1, p_2) receive an expected utility strictly inferior to v^*. This cannot happen in equilibrium. ■

Proof of Proposition 5. In equilibrium it must be the case that

$$v^U\left(0,\left(p,p\right)\right) = v^U\left(1,\left(p,p\right)\right) = v^U\left(2,\left(p,p\right)\right)$$

and the last equality implies

$$\Pr\left(\chi_1 = 0|\left(p,p\right)\right) = \Pr\left(\chi_2 = 0|\left(p,p\right)\right). \tag{17}$$

Since

$$\Pr\left(\chi_i = 0|\left(p,p\right)\right) = \Pr\left(\chi_i = 0, \chi_{-i} = 0|\left(p,p\right)\right) + \Pr\left(\chi_i = 0, \chi_{-i} = 1|\left(p,p\right)\right)$$

the condition boils down to

$$\Pr\left(\chi_1 = 0, \chi_2 = 1|\left(p,p\right)\right) = \Pr\left(\chi_1 = 1, \chi_2 = 0|\left(p,p\right)\right)$$

which in turn is equivalent to

$$\Pr\left(\chi_1 = 0, \chi_2 = 1 \text{ and } \left(p,p\right)\right) = \Pr\left(\chi_1 = 1, \chi_2 = 0 \text{ and } \left(p,p\right)\right). \tag{18}$$

How can this be achieved? We start observing that in each equilibrium, whenever the prices for the risky assets are (p, p) with $0 < p < \frac{1}{R}$ and the realization of $\widetilde{\chi}$ is different from $(0, 0)$ then the optimal demand of an informed manager y is unique and given by:

$$d^y\left(\left(p,p\right)|\chi\right) = \begin{bmatrix} \chi_1\chi_2 \\ \left(1-\chi_1\right)\chi_2 \\ \chi_1\left(1-\chi_2\right) \end{bmatrix} \tag{19}$$

When $(\chi_1, \chi_2) = (0, 0)$ and the prices are (p, p) then informed managers are indifferent between the three vectors $(0, 1, 0)$, $(0, 0, 1)$ and $(0, 1, 1)$. Suppose

that whenever $\chi_1 = \chi_2 = 0$ and prices are (p,p) then a fraction α of informed managers demands asset 1 and a fraction β demands asset 2, with $\alpha \in [0,1]$, $\beta \in [0,1]$, $\alpha + \beta \in [1,2]$. For example, if $\alpha = \beta = 1$ this means that all informed managers submit the demand vector $(0,1,1)$ when $\chi = (0,0)$ and the price vector is (p,p). The aggregate demand of informed managers is therefore

$$\int_{y \in N^I} d^y \left((p,p) | \, (0,0) \right) dy = \begin{bmatrix} 0 \\ \alpha N^I \\ \beta N^I \end{bmatrix} \tag{20}$$

We have now to find the subset in the space $\left[\underline{b}, \overline{b}\right]^2 \times \{0,1\}^2$ for which $p^e(b,\chi) = (p,p)$. We will go through the four possible realizations of χ.

Case $\chi = (0,0)$.
We first show that if $b_1 < \underline{b} + \alpha N^I$ or $b_2 < \underline{b} + \beta N^I$ then prices are fully revealing, i.e. $p^e(b,\chi) = \left(\frac{1}{R}, \frac{1}{R}\right)$. Suppose not. Take $b_1 < \underline{b} + \alpha N^I$ and suppose that the price vector is not fully revealing, so that $p^e(b,\chi) = (p,p)$. Then there must be vectors $\chi' = (1, \chi_2')$ and b' such that the excess demand is the same as at (b,χ) when the price pair is (p,p). This implies

$$\left(b_1 - \alpha N^I, b_2 - \beta N_I\right) = \left(b_1', b_2' - N^I\left(1 - \chi_2'\right)\right).$$

In particular this requires $b_1' = b_1 - \alpha N^I < \underline{b}$, which is impossible. A similar reasoning applies when $b_2 < \underline{b} + \beta N^I$.

Next we show that if the two inequalities $b_1 \geq \underline{b} + \alpha N^I$ and $b_2 \geq \underline{b} + \beta N^I$ are satisfied then the price is not fully revealing, i.e. $p^e(b,\chi) = (p,p)$. Suppose not. Then the price vector is fully revealing, so that $p^e(b,\chi) = \left(\frac{1}{R}, \frac{1}{R}\right)$. Now consider the vectors $b' = \left(b_1 - \alpha N^I, b_2 - \beta N^I\right)$ and $\chi' = (1,1)$. It is easy to check that at (b', χ') the excess demand when the prices are $\left(\frac{1}{R}, \frac{1}{R}\right)$ is the same as at (b,χ). This is a contradiction, since $\left(\frac{1}{R}, \frac{1}{R}\right)$ can be an equilibrium price vector only when $\chi = (0,0)$.

We conclude that when $\chi = (0,0)$ we should have $p^e(\chi, b) = (p,p)$ whenever $(b_1, b_2) \in \left[\underline{b} + \alpha N^I, \overline{b}\right] \times \left[\underline{b} + \beta N^I, \overline{b}\right]$. In particular, the probability of the event

$$E_{pp}^{00} = (\chi = (0,0) \text{ and } (p,p) \text{ observed})$$

is given by

$$\Pr\left(E_{pp}^{00}\right) = (1 - q_1)(1 - q_2)\frac{\Delta b - \alpha N^I}{\Delta b}\frac{\Delta b - \beta N^I}{\Delta b} =$$

$$(1 - q_1)(1 - q_2)(1 - \alpha r)(1 - \beta r).$$

Case $\chi = (0,1)$.

We first show that if $b_1 < \underline{b} + N^I$ or $b_2 > \overline{b} - \beta N^I$ then the price is fully revealing, i.e. $p^e(b,\chi) = \left(\frac{1}{R}, 0\right)$. Suppose first that $b_1 < \underline{b} + N^I$ and the price vector is not fully revealing, i.e. $p^e(b,\chi) = (p,p)$. To make sure that $\chi_1 = 0$ is not revealed, there must be a pair (b', χ') with $\chi' = (1, \chi'_2)$ such that the excess demand is the same as at (b,χ). This requires

$$\left(b_1 - N^I, b_2\right) = \left(b'_1, b'_2 - \left(1 - \chi'_2\right) N^I\right)$$

and in particular $b'_1 = b_1 - N^I < \underline{b}$, which is impossible. Suppose now $b_2 > \overline{b} - \beta N^I$. There must be a pair (b', χ') with $\chi' = (\chi'_1, 0)$ such that the excess demand is the same as at (b,χ). If $\chi'_1 = 0$ then the condition becomes

$$\left(b_1 - N^I, b_2\right) = \left(b'_1 - \alpha N^I, b'_2 - \beta N^I\right)$$

or $b'_2 = b_2 + \beta N^I > \overline{b}$, which is impossible. If $\chi'_1 = 1$ then the condition becomes

$$\left(b_1 - N^I, b_2\right) = \left(b'_1, b'_2 - N^I\right)$$

or $b'_2 = b_2 + N^I > \overline{b}$, which is also impossible.

We next show that if $b_1 \geq \underline{b} + N^I$ and $b_2 \leq \overline{b} - \beta N^I$ then the price vector is not fully revealing, that is $p^e(b,\chi) = (p,p)$. Suppose not, so that the price vector is revealing and $p^e(b,\chi) = \left(\frac{1}{R}, 0\right)$. Suppose first $b_1 \geq \underline{b} + N^I$ and consider the vectors $\chi' = (1,1)$ and $b' = \left(b_1 - N^I, b_2\right)$. Then, at $\left(\frac{1}{R}, 0\right)$ the excess demand is the same as at (b,χ), a contradiction. Similarly, if $b_2 \leq \overline{b} - \beta N^I$ consider the vectors $\chi' = (0,0)$ and $b' = \left(b_1 - (1-\alpha) N^I, b_2 + \beta N^I\right)$. Again the excess demand is the same at $\left(\frac{1}{R}, 0\right)$, a contradiction.

We conclude that when $\chi = (0,1)$ we should have $p^e(\chi, b) = (p,p)$ whenever $(b_1, b_2) \in \left[\underline{b} + N^I, \overline{b}\right] \times \left[\underline{b}, \overline{b} - \beta N^I\right]$. The probability of the event

$$E^{01}_{pp} = (\chi = (0,1) \text{ and } (p,p) \text{ observed})$$

is given by

$$\Pr\left(E^{01}_{pp}\right) = (1 - q_1)\, q_2 \left(\frac{\Delta b - N^I}{\Delta b}\right) \frac{\Delta b - \beta N^I}{\Delta b}$$

$$= (1 - q_1)\, q_2\, (1 - r)\, (1 - \beta r)$$

Case $\chi = (1,0)$.

This case is symmetric to the previous one, so applying the same reasoning we can conclude that when $\chi = (0,1)$ we should have $p^e(b,\chi) = (p,p)$ whenever $(b_1,b_2) \in \left[\underline{b}, \overline{b} - \alpha N^I\right] \times \left[\underline{b} + N^I, \overline{b}\right]$. The probability of the event

$$E_{pp}^{10} = (\chi = (1,0) \text{ and } (p,p) \text{ observed})$$

is

$$\Pr\left(E_{pp}^{10}\right) = q_1(1-q_2)(1-\alpha r)(1-r).$$

Case $\chi = (1,1)$.
Again, applying the same reasoning as above we can conclude that when $\chi = (1,1)$ we should have $p^e(b,\chi) = (p,p)$ whenever $(b_1,b_2) \in \left[\underline{b}, \overline{b} - \alpha N^I\right] \times \left[\underline{b}, \overline{b} - \beta N^I\right]$. The probability of this event

$$E_{pp}^{11} = (\chi = (1,1) \text{ and } (p,p) \text{ observed})$$

is

$$\Pr\left(E_{pp}^{11}\right) = q_1 q_2 (1-\alpha r)(1-\beta r)$$

After examining the 4 cases we can conclude that the probability of observing (p,p) in equilibrium is strictly positive and condition (18) becomes equivalent to

$$(1-q_1)\,q_2\,(1-\beta r) = q_1\,(1-q_2)\,(1-\alpha r)$$

or

$$\frac{(1-q_1)\,q_2}{(1-q_2)\,q_1} = \frac{1-\alpha r}{1-\beta r}. \tag{21}$$

An equilibrium exists if we can find feasible values α and β such that (21) is satisfied Since feasibility requires $1 \geq \alpha \geq 0$, $1 \geq \beta \geq 0$ and $2 \geq \alpha + \beta \geq 1$, the lowest possible value attainable by the ratio $\frac{1-\alpha r}{1-\beta r}$ is $1-r$, while the highest possible value is $\frac{1}{1-r}$. Since $q_2 > q_1$ the LHS in (21) is strictly greater than 1. We conclude that a solution exists if

$$\frac{(1-q_1)\,q_2}{(1-q_2)\,q_1} \leq \frac{1}{1-r}$$

or

$$r q_2 (1-q_1) \geq q_2 - q_1.$$

The probability of failure for each risky asset condition on observing (p,p) is

$$f = \frac{\Pr\left(E_{pp}^{11}\right) + \Pr\left(E_{pp}^{10}\right)}{\Pr\left(E_{pp}^{11}\right) + \Pr\left(E_{pp}^{10}\right) + \Pr\left(E_{pp}^{01}\right) + \Pr\left(E_{pp}^{00}\right)} \tag{22}$$

and it can be computed from the formulas above. The price p has to satisfy

$$p = \frac{1-f}{R + \left(f^2 - (1-f)\right)\beta\omega\delta\frac{W^U}{\gamma}} \tag{23}$$

so that $v^U\left(i,(p,p)\right) = v^U\left(0,(p,p)\right)$ for $i = 1,2$. ■

Proof of Proposition 6. In any simple equilibrium the structure of the non-revealing region conforms to the one described in the proof of Proposition 5. We now show how this structure can be embedded in a rational expectations equilibrium compatible with excess demand. We will first describe the price function $p^e\left(b,\chi\right)$ for each pair (b,χ) and then describe the demand and allocation functions supporting the equilibrium.

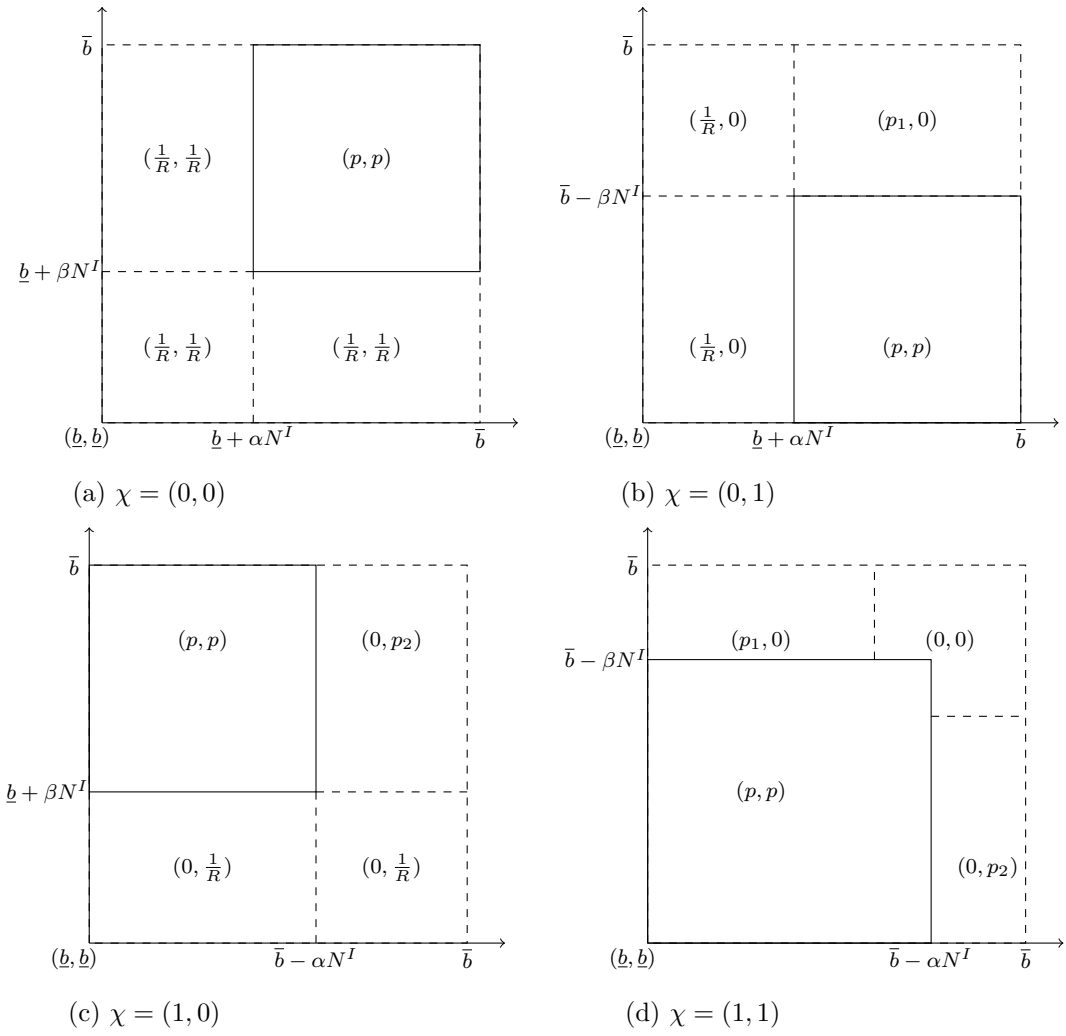
The price function.
For each of the possible values of the pair (χ_1,χ_2) we will show how the space $\left[\underline{b},\overline{b}\right]^2$ is partitioned and the values taken by the price function.

Case $\chi = (0,0)$. The prices are non-revealing when both supply shocks are sufficiently high, and they are fully revealing otherwise. Figure 1a shows the price function for this case. It is never the case that the value of one asset is revealed while the other is not, as this may happen only when $\chi_i = 1$ for some i. The value of p is given by (22) and (23).

Case $\chi = (0,1)$. In this case there is an area in which the value of risky asset 2 is revealed but the value of risky asset 1 is not. Figure 1b shows these regions. For this to be possible, the excess demands in the region must be indistinguishable from excess demands when $\chi = (1,1)$ and the true value of risky asset 2 is revealed. The value of p_1 is given by

$$p_1 = \frac{1-q_1}{R + (2q_1 - 1)\frac{\beta\omega\delta}{\gamma}W^U}.$$

Case $\chi = (1,0)$. Figure 1c shows the values of price function for this case. This case is symmetric to the previous one.



(a) $\chi = (0, 0)$ (b) $\chi = (0, 1)$

(c) $\chi = (1, 0)$ (d) $\chi = (1, 1)$

Figure 1: Price function at any (χ_1, χ_2)

Now there is an area in which it is revealed that asset 1 is defaulting while there is uncertainty on asset 2. The value of p_2 is given by

$$p_2 = \frac{1 - q_2}{R + (2q_2 - 1)\frac{\beta\omega\delta}{\gamma}W^U}$$

Case $\chi = (1,1)$. This is the most complex case. Figure 1d shows the revelation and non-revelation regions for this case. Besides the area of non-revelation there are two areas of partial revelation, in which either asset 1 or asset 2 is revealed in default, as well as an area in which there is full revelation and both assets are revealed in default.

The values of p_1 and p_2 are determined as follows. In equilibrium it has to be the case that

$$(1 - f_i)\left(\frac{\gamma}{p_i} + \beta\omega\delta W^U\right) = \gamma R + f_i\beta\omega\delta W^U.$$

Observe now that

$$\begin{aligned} f_1 = \Pr\left(\chi_1 = 1 | \left(p_1, 0\right)\right) &= \frac{\Pr\left(\chi_1 = 1 | \left(p_1, 0\right)\right)}{\Pr\left(\chi_1 = 1 | \left(p_1, 0\right)\right) + \Pr\left(\chi_1 = 0 | \left(p_1, 0\right)\right)} \\ &= \frac{q_1\left(1 - \beta r\right)\left(1 - r\right)}{q_1\left(1 - \beta r\right)\left(1 - r\right) + \left(1 - q_1\right)\left(1 - \beta r\right)\left(1 - r\right)} = q_1. \end{aligned}$$

A similar reasoning establishes that $f_2 = q_2$.

The demand functions.
The demand functions for informed managers supporting this equilibrium are the ones described in Proposition 5. For uninformed managers the demand is described as follows:

- when the prices are (p, p) or $\left(\frac{1}{R}, \frac{1}{R}\right)$the demand is $d^y = (1, 1, 1)$;
- when the prices are $(p_1, 0)$ or $\left(\frac{1}{R}, 0\right)$ the demand is $d^y = (1, 1, 0)$;
- when the prices are $(0, p_2)$ or $\left(0, \frac{1}{R}\right)$ the demand is $d^y = (1, 0, 1)$;
- for all other pairs $(\widehat{p}_1, \widehat{p}_2)$ the demand is $d^y = (1, 0, 0)$.

The optimality of the demand functions for the informed is immediate. For the uninformed, given the equilibrium price function above optimality is

clear when prices are observed in equilibrium. If a pair $(\widehat{p}_1, \widehat{p}_2)$ is not observed in equilibrium we specify that uninformed managers assume that the risky assets are defaulting and demand the riskless asset only.

The feasible allocation rule.

If d^y has a single non-zero element i then feasibility requires $x_i = 1$ and $x_j = 0$ for $j \neq i$. Suppose d^y has multiple non-zero elements and enumerate the elements as $\{i_1, \ldots, i_m\}$. Then x_{i_k} is defined recursively as

- $x_{i_m} = \frac{b_{i_m} - \xi_{i_m}}{D_{i_m} - \xi_{i_m}}$;
- $x_{i_k} = \min\left\{ \frac{b_{i_k} - \xi_{i_k}}{D_{i_k} - \xi_{i_k}}, 1 - \sum_{j=k+1}^{m} x_{i_j} \right\}$;
- $x_{i_1} = 1 - \sum_{j=2}^{m} x_{i_j}$.

To see how the allocation rule works, consider an equilibrium in which $\beta = 1 - \alpha$. If $\chi = (0, 0)$ and the realized price pair is (p, p) then all uninformed traders have a demand vector $(1, 1, 1)$, while a fraction αN^I of informed managers has demand vector $(0, 1, 0)$ and a fraction $(1 - \alpha) N^I$ has demand vector $(0, 0, 1)$. We therefore have $\xi_1 = \alpha N^I$, $\xi_2 = (1 - \alpha) N^I$, $D_1 = N^U + \alpha N^I$ and $D_2 = N^U + (1 - \alpha) N^I$. The amount of asset 2 allocated is

$$N^U \frac{b_2 - (1 - \alpha) N^I}{N^U} + (1 - \alpha) N^I = b_2,$$

so that all the supply is allocated. For asset 1 we can check that

$$\frac{b_1 - \xi_1}{D_1 - \xi_1} < 1 - \frac{b_2 - \xi_2}{D_2 - \xi_2}$$

so that the quantity allocated is

$$N^U \frac{b_1 - \alpha N^I}{N^U} + \alpha N^I = b_1,$$

and again al the supply is allocated. The rest of the uninformed managers receive the riskless asset. Other cases are treated similarly. ■

Proof of Proposition 7. Let η_t be the probability that a manager is informed at time t and let $\zeta_t(b, \chi)$ be the equilibrium probability that an uninformed manager is fired at time t when the realization of the random

variable is (b, χ). The probability is zero when (b, χ) is such that the price vector is fully revealing and it is strictly positive otherwise.

In a stationary equilibrium an investor is separated from an informed manager only for exogenous reasons. Thus, the mass of informed managers losing the job in every period is

$$A^I = (1 - \delta) N^I \tag{24}$$

and it does not depend on t or the realization (b, χ). The measure of uninformed managers who lose their job at period t depends on (b_t, χ_t) and it is given by

$$A_t^U (b_t, \chi_t) = ((1 - \delta) + \delta \zeta_t (b_t, \chi_t)) N^U \tag{25}$$

Thus at any given time t the mass of available positions is

$$A_t (b_t, \chi_t) = (1 - \delta) N^I + ((1 - \delta) + \delta \zeta_t (b_t, \chi_t)) N^U \tag{26}$$

To maintain the values N^I and N^U constant, the mass of informed managers who lose the job must be replaced by an equal mass of informed managers who are hired, and the same is true for uninformed managers. Thus we must have

$$\mu^* Z^I = A^I$$
$$\mu^* Z_t^U (b_t, \chi_t) = A_t^U (b_t, \chi_t)$$

which yields

$$\frac{Z_t^U (b_t, \chi_t)}{Z^I} = \frac{A_t^U (b_t, \chi_t)}{A^I} = \frac{((1 - \delta) + \delta \zeta_t (b_t, \chi_t)) N^U}{(1 - \delta) N^I}.$$

The probability of getting an informed manager at time t is

$$\epsilon_t = \frac{Z^I}{Z^I + Z_t^U}$$

The equilibrium condition is therefore that it is always the case, for each possible history, that the belief η_t following a history in which the manager picked the 'right' investment and was not revealed uninformed by the exogenous signal is such that $\eta_t \geq \epsilon_t$, i.e. the probability of being informed assigned to a manager making no mistakes is higher than the probability of hiring an informed manager on the labor market.

In equilibrium, the Bayes rule implies

$$\eta_{t+1} = \frac{\eta_t}{\eta_t + (1 - \zeta_t)(1 - \eta_t)} \tag{27}$$

for a manager who has not been hired at the end of period t, where the dependence of ζ_t on (b_t, χ_t) has been omitted for simplicity. For a newly hired manager we have $\eta_t = \epsilon_{t-1}$. By (27) we have

$$\eta_{t+1} = \frac{\epsilon_{t-1}}{\epsilon_{t-1} + (1-\zeta_t)(1-\epsilon_{t-1})}.$$

We first prove that $\eta_{t+1} > \epsilon_t$. Note that this is equivalent to proving

$$\frac{1-\epsilon_t}{\epsilon_t} > \frac{1-\epsilon_{t-1}}{\epsilon_{t-1}}(1-\zeta_t) \tag{28}$$

Since $\epsilon_t = Z^I / \left(Z^I + Z_t^U\right)$ we have,

$$\frac{1-\epsilon_t}{\epsilon_t} = \frac{Z_t^U}{Z^I} = \frac{((1-\delta) + \delta\zeta_t) N^U}{(1-\delta) N^I}$$

Using an analogous expression for $\frac{1-\epsilon_{t-1}}{\epsilon_{t-1}}$ we can write inequality (28) as

$$\zeta_t > \delta\zeta_{t-1}(1-\zeta_t).$$

Since $\zeta_{t-1} \leq 1$, a sufficient condition for the inequality to be satisfied is $\zeta_t > \delta(1-\zeta_t)$, or $\zeta_t > \frac{\delta}{1+\delta}$. Since $\zeta_t^U \geq 1-\omega$, the inequality is satisfied by Assumption 4, and therefore $\eta_{t+1} > \epsilon_t$.

When the manager has been employed for more than one period the reasoning is similar. Suppose the manager was hired at the end of $t'-1$ and has not made any mistake and σ_t^y for $t \geq t'$ has been always 0. This implies that the updated belief of the investor at the end of any time period t is not less than his belief at the beginning of t for $t \geq t'$ and therefore it is also greater than his initial belief at t'. That is

$$\eta_{t+1} = \frac{\eta_t}{\eta_t + (1-\zeta_t^U)(1-\eta_t)} \geq \eta_{t'} = \frac{\epsilon_{t'-1}}{\epsilon_{t'-1} + (1-\zeta_{t'}^U)(1-\epsilon_{t'-1})}$$

for any $t \geq t'$. Hence, a sufficient condition for $\eta_{t+1} > \epsilon_t$ is having

$$\frac{\epsilon_{t'-1}}{\epsilon_{t'-1} + (1-\zeta_{t'}^U)(1-\epsilon_{t'-1})} \geq \epsilon_t$$

But by the same argument, this inequality holds by assumption 4. ■

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